Preferential Attachment in Online Networks: Measurement and Explanations

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Networks Are Everywhere
Power Laws – Scale Free Networks

\[ C(d) \sim d^{-\gamma} \]
Preferential Attachment Model

\[ P(\{A, i\}) \sim d(i) \]
Linear vs Nonlinear Preferential Attachment

\[ f(d) \sim 1 \quad \text{Erdős–Rényi model [1]} \]
\[ f(d) \sim d^\beta, \ 0 < \beta < 1 \quad \text{Sublinear model [2]} \]
\[ f(d) \sim d \quad \text{Barabási–Albert model [3]} \]
\[ f(d) \sim d^\beta, \ \beta > 1 \quad \text{Superlinear model [4]} \]

Erdős–Rényi Model (1959)

\[ P(\{i, j\}) = p \]

- Every edge equiprobable
- No structure
- Binomial degree distribution

\[ C(d) \sim p^d (1 - p)^{|V| - 1 - d} \]

Barabási–Albert Model (1999)

$P(\{A, i\}) \sim d(i)$

- Generative model
- Scale-free network
- Power law degree distribution

$C(d) \sim d^{-\gamma}$

Sublinear Model

\[ P(\{A, i\}) \sim d(i)^\beta \]

\( 0 < \beta < 1 \)

- Stretched exponential degree distribution

[1, Eq. 94]

Superlinear Model

\[ P(\{A, i\}) \sim d(i)^\beta \]
\[ \beta > 1 \]

- A single node attracts 100% of edges asymptotically
- Power law degree distribution in the pre-asymptotic regime

Temporal Network Data

Network at time $t_1$
Degrees $d_1(u)$

Added edges
Degrees $d_2(u)$

Network at time $t_2$
Degrees $d_1(u) + d_2(u)$

Hypothesis: $d_2 = \alpha d_1^\beta$
Empirical Measurement of $\beta$

\[ d_2 = e^{\alpha} \left( 1 + d_1 \right)^{\beta} - \lambda \]

Find $(\alpha, \beta)$ using least squares:

\[ \min_{\alpha, \beta} \sum_{u \in V} \left( \alpha + \beta \ln[1 + d_1(u)] - \ln[\lambda + d_2(u)] \right)^2 \]

\[ \varepsilon = \exp \left\{ \sqrt{1 / |V|} \sum_{u \in V} (\alpha + \beta \ln[1 + d_1(u)] - \ln[\lambda + d_2(u)])^2 \right\} \]
Example Network: Facebook Wall Posts

Description: User–user wall posts
Format: Edges are directed
Edge weights: Multiple edges are possible
Metadata: Edges have timestamps
Size: 63,891 vertices
Volume: 876,993 edges
Average degree: 27.45 edges / vertex
Maximum degree: 2,696 edges

HTTP://KONECT.UNI-KOBLENZ.DE/NETWORKS/FACEBOOK-WOSN-WALL
Facebook Wall Post Preferential Attachment

Regularized number of new edges \((\lambda + d_2(u))\)

**Data**

\[
0.1181 \times x^{0.9520}
\]

\[
\varepsilon = 5.3918
\]
## Network Categories

<table>
<thead>
<tr>
<th></th>
<th>Interaction Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social network</td>
<td>user–user</td>
</tr>
<tr>
<td>Rating network</td>
<td>user–item</td>
</tr>
<tr>
<td>Communication network</td>
<td>user–user</td>
</tr>
<tr>
<td>Folksonomy</td>
<td>person–tag/item</td>
</tr>
<tr>
<td>Wiki edit network</td>
<td>editor–article</td>
</tr>
<tr>
<td>Explicit interaction network</td>
<td>person–person</td>
</tr>
<tr>
<td>Implicit interaction network</td>
<td>person–item</td>
</tr>
</tbody>
</table>
Comparison

- Social network: $\beta < 1$
- Rating network: $\beta < 1$
- Communication network: $\beta < 1$
- Folksonomy: $\beta < 1$
- Wiki edit network
- Explicit interaction network: $\beta > 1$
- Implicit interaction network: $\beta > 1$
Thank You

Datasets available at:

http://konect.uni-koblenz.de/

Read our blog:

https://blog.west.uni-koblenz.de/2013-04-29/
the-linear-preferential-attachment-assumption-and-its-generalizations/