Diversity Dynamics in Online Networks

Jérôme Kunegis¹  Sergej Sizov¹  Felix Schwagereit¹  Damien Fay²

¹ University of Koblenz–Landau, Germany  ² University College Cork, Ireland

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Everyone likes good things:
Or even better: Diversity!
Structural Diversity
(1) Length of paths

Diversity

“Large” world

No diversity

Small world
90-percentile effective diameter $\delta_{0.9}$

![Hop plot](image)

- $\delta_m = 7.48$
- $\delta_{0.5} = 6.79$
- $\delta_{0.9} = 9.98$
- $\delta = 24$
"The World Gets Smaller"

(Leskovec, Kleinberg & Faloutsos 2007)
Outline

(A) How can structural diversity be measured?
(B) How does diversity change?
(A) How to Measure Diversity in a Network?

(1) Length of paths
(2) Numbers of neighbors
(3) Size of communities
(4) Random walks
(5) Controllability
(2) Number of neighbors

\[ d(i) \approx d(j) \quad \text{Diversity} \]

\[ d(i) \ll d(j) \quad \text{No diversity} \]
Gini Coefficient

Share of edges

Share of nodes with smallest degrees

G = 39.7%
"The Rich Get Richer"

(Barabási & Albert 1999)
(3) Size of communities

Diversity

No diversity
Fractional Rank

Spectrum of the graph \(= \{\lambda_1, \lambda_2, \lambda_3, \ldots \}\)

\[
\text{rank}_F = \sum_k \left(\frac{\lambda_k}{\lambda_1}\right)^2 = \left(\frac{\|A\|_F}{\|A\|_2}\right)^2
\]
“Eigenvector Preferential Attachment”

\[ U_{i1} U_{j1} > \frac{\lambda_1}{2|E|} \]
(4) Random walks

Diversity

\[ P_{\text{ret}}(L) \text{ large} \]

No diversity

\[ P_{\text{ret}}(L) \text{ small} \]
Weighted Spectral Distribution

\[ P_{\text{ret}}(L) = \sum_{(i, j, \ldots, k)} (d(i) d(j) \ldots d(k))^{-1} \]
\[ = \text{tr}(N^L) \]
\[ = \sum_k \lambda_k^L \]

where \( \lambda_k \) are eigenvalues of \( N = D^{-1/2} A D^{-1/2} \).

Here: Use \( L = 4 \) and \( k \leq R \)
Eigenvalues of $\mathbb{N}$

Frequency

Eigenvalues ($\lambda_k$)

Time 1

Time 2
“Random Walks Arrive Less Often”

Weighted spectral distr. (WSD)

Volume (|E|) [edges] $\times 10^5$
(5) Controllability

Diversity

No diversity

(Liu, Slotine & Barabási 2011)
Find a maximal directed 2-matching

$$\#\text{Knobs needed} = |V| - \max |M|$$
"Networks Get Easier to Control"
(B) Experiments

20 networks from konect.uni-koblenz.de

9 authorship, 3 communication, 3 social, 3 interaction, 1 rating, 1 physical

<table>
<thead>
<tr>
<th>Measure</th>
<th>Diversity decreasing</th>
<th>No diversity trend</th>
<th>Diversity increasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Diameter</td>
<td>12</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>(2) Gini coefficient</td>
<td>13</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>(3) Fractional rank</td>
<td>10</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>(4) Weighted spectral distribution</td>
<td>12</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>(5) Controllability</td>
<td>15</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>
Thank You

Jérôme Kunegis
Sergej Sizov
Felix Schwagereit
Damien Fay

University of Koblenz–Landau, Germany
University College Cork, Ireland

konect.uni-koblenz.de
Questions

Did you try the power law exponent instead of the Gini coefficient?  
→ Yes, but see (Kunegis & Preusse 2012)

Did you try the absolute value instead of the square in rank_F?  
→ Yes, it leads to the nuclear norm instead of the Frobenius norm

Isn't it hard to find a maximal directed 2-matching?  
→ It takes a runtime of O(|V|^{1/2} |E|)

How is the approximation using only R eigenvalues for the WSD justified?  
→ By observing that all eigenvalues shrink
References


Credits

http://www.shewearsshortshorts.com/2012/01/downside.html
https://twitter.com/#!/justinbieber
http://www.iconspedia.com/icon/nerd-4255.html
http://hk.digikey.com/1/3/index1227.html