DAI-Labor TU Berlin

Spectral Analysis of Signed Graphs for Clustering, Prediction and Visualization

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Summary

The **Laplacian matrix** applies to **signed graphs**

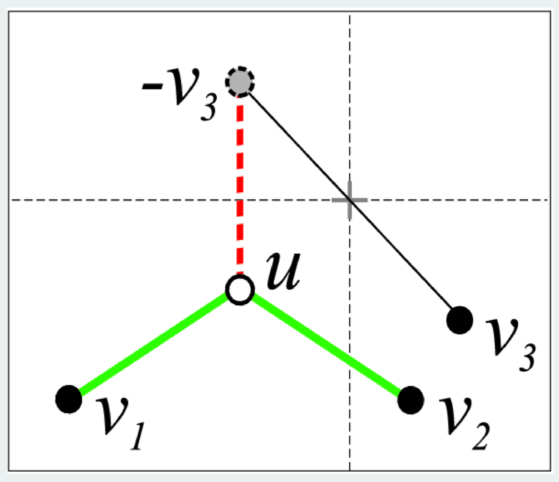
- The Laplacian spectrum denotes **graph balance**
- The Laplacian implements **antipodal proximity**
- The Laplacian implements **signed cuts**
- The Laplacian models negation as **inversion of electrical potential**

Signed Graph Drawing

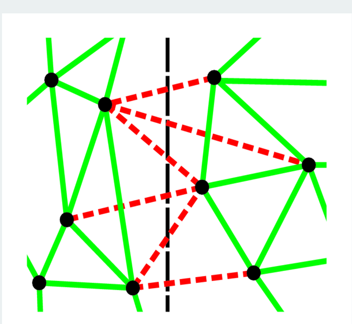
- Place node near positive neighbors
- Place node far from negative neighbors

u = (1/3) (v1 + v2 − v3)

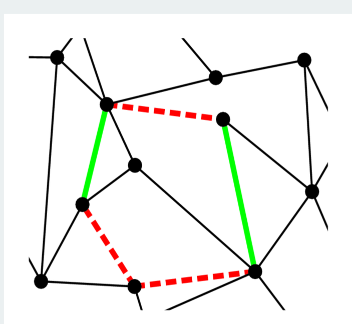
- Leads to lower eigenvectors of signed Laplacian
  L = **D** − **A**



Balance and Conflict



Balance

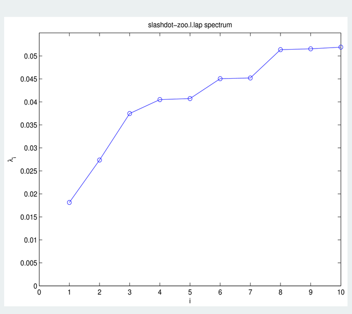


Conflict

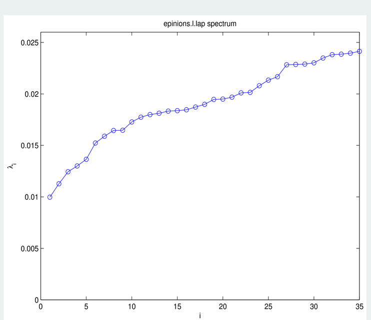
A network is balanced when:

- There is a 2-clustering consistent with edge signs
- All cycles have an odd number of negative edges

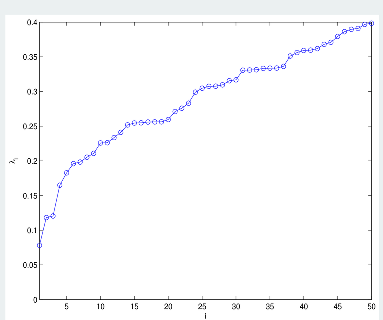
Signed Spectrum of Large Networks



Slashdot Zoo

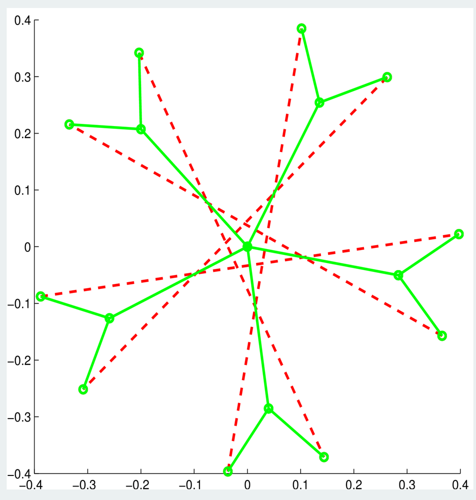


Epinions

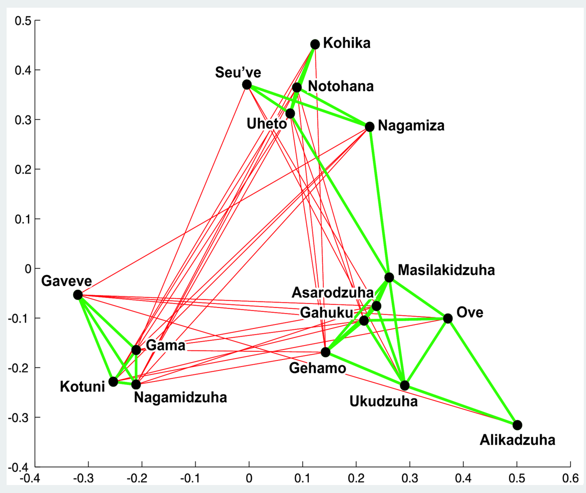


MovieLens

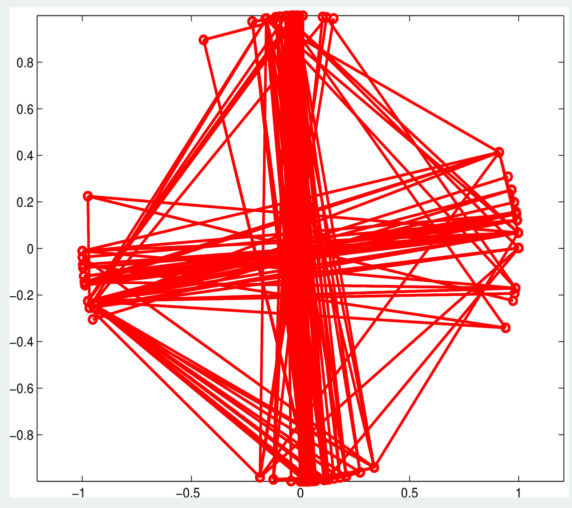
Examples – Signed Graphs



Synthetic example



Tribal groups of the Eastern Central Highlands of New Guinea
Friends (‘Rova’) and Foes (‘Hina’)



Wikipedia reverts on controversial article ‘Criticism of Prem Rawat’

The Signed Graph Laplacian

$\mathbf{A} = \{0, +1, -1\}^{n\times n}$	Adjacency matrix
$D_{ii} = \Sigma_j \lvert A_{ij}\rvert$	Degree matrix
$\mathbf{L} = \mathbf{D} - \mathbf{A}$	Signed Laplacian

- Positive semidefinite: $x^T\mathbf{L}x = \Sigma_{ij} \lvert A_{ij}\rvert(x_i - \mathrm{sgn}(A_{ij})\, x_j)^2 \geq 0$
- Positive definite when the network is unbalanced
- Smallest eigenvalue denotes conflict: It is zero when the network is balanced and larger when there is conflict

Signed Spectral Clustering

- Communities in signed graphs:

  - Positive edges inside communities
  - Negative edges between communities

- Minimize the signed ratio cut:

  min (2 pos(X,Y) − neg(X,X) − neg(Y,Y)) ($|X|^{-1} + |Y|^{-1}$)

- pos(X,Y) counts positive edges between X and Y
- neg(X,Y) counts negative edges between X and Y
- Relaxation gives the lower eigenvalues of **L**