Exercises for Advances in Theoretical Computer Science
Exercise Sheet 2

Due at 08.11.2021, 10:00 s.t.

Remark: To ease the task for exercises involving Turing machines, you are allowed to use Turing machines introduced in the lecture, in the book by Erk and Priese and in previous exercises. Additionally you are allowed to define and use “auxiliary Turing machines”. Definitions in flow chart notation are favoured over formal definitions.

Exercise 2.1

Let $\Sigma$ be arbitrary. Define deterministic Turing machines $M_Y$, $M_N$ which for every $w \in \Sigma^*$ compute as follows:

\[ s, \#w\# \frac{}{\vdash_{M_Y} h, \#Y\#} \]
\[ s, \#w\# \frac{}{\vdash_{M_N} h, \#N\#} . \]

Exercise 2.2

Let $\Sigma = \{a, b\}$. Define a 2-tape deterministic Turing machine $M_{\text{pal}}$ which recognizes palindroms over $\Sigma$, i.e. it computes as follows:

\[ s, \#w\#, \# \frac{}{\vdash_{M_{\text{pal}}} h, \#Y\#, \#} \text{ if } w \in \Sigma^* \text{ is a palindrome} \]
\[ s, \#w\#, \# \frac{}{\vdash_{M_{\text{pal}}} h, \#N\#, \#} \text{ if } w \in \Sigma^* \text{ is not a palindrome}. \]

Exercise 2.3

Let $\Sigma = \{\|\}$. Define a 3-tape deterministic Turing machine $M_{\text{mul}}$ that calculates the product of the numbers $n_1, n_2$ in unary notation.

\[ s, \#|n_1|\#|n_2|\#, \# \frac{}{\vdash_{M_{\text{mul}}} h, \#|n_1 \times n_2|\#, \#, \#}. \]
Exercise 2.4

Prove or refute the following statements:

I) The union of two decidable languages is decidable.
II) The intersection of two decidable languages is decidable.
III) The concatenation of two decidable languages is decidable.
IV) The complement of a decidable language is decidable.
V) The union of two recursively enumerable languages is recursively enumerable.
VI) The intersection of two recursively enumerable languages is recursively enumerable.
VII) The concatenation of two recursively enumerable languages is recursively enumerable.
VIII) The complement of a recursively enumerable language is recursively enumerable.

If you want to submit solutions, please do so until 08.11.2021, 10:00 s.t. via e-mail (with “Homework ACT CS” in the subject) to dpeuter@uni-koblenz.de.