Exercises for Advances in Theoretical Computer Science  
Exercise Sheet 4  
Due at 22.11.2021, 10:00 s.t.

Remark: For writing pseudocode, use the language that is defined at the home page of the exercise. For register machine programs, you are allowed to use the instructions defined at the home page of the exercise.

Exercise 4.1
Let $P$ be the following LOOP-program.

```
loop $x_1$ do
    $x_3 := x_3 + 1$
end;
// (1)
loop $x_3$ do
    $x_2 := x_2 + x_3$
end;
// (2)
x_3 := 0
```

a) Fill in the following table with the values of the registers $x_1, x_2, x_3$ at points (1) and (2) in the program: (i) for input 3; (ii) for input 5.

<table>
<thead>
<tr>
<th>Input 3</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input 5</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Which is the output of $P$ for input 3? Which is the output of $P$ for input 5?

c) Which function $f : \mathbb{N} \rightarrow \mathbb{N}$ is computed by $P$?

Exercise 4.2
Let $\text{fac} : \mathbb{N} \rightarrow \mathbb{N}$ be defined, for every $n \in \mathbb{N}$, by $\text{fac}(n) := n!$.

a) Write a pseudocode program that computes $\text{fac}$.

b) Write a LOOP program that computes $\text{fac}$.

Exercise 4.3
Let the function $\text{fib} : \mathbb{N} \rightarrow \mathbb{N}$ be defined, for every $n \in \mathbb{N}$, by

$$
\text{fib}(n) := \begin{cases} 
1 & \text{if} \ n = 0 \\
1 & \text{if} \ n = 1 \\
\text{fib}(n - 1) + \text{fib}(n - 2) & \text{otherwise.}
\end{cases}
$$

a) Write a pseudocode program that computes $\text{fib}$.

b) Write a WHILE program that computes $\text{fib}$. 


**Exercise 4.4**

a) Let $\text{DIV}_{10} : \mathbb{N} \rightarrow \mathbb{N}$ be defined, for every $n \in \mathbb{N}$, by $\text{DIV}_{10}(n)$ is the result of the integer division of $n$ by 10.

   I) Write a pseudocode program that computes $\text{DIV}_{10}$.
   II) Write a GOTO program that computes $\text{DIV}_{10}$.

b) Let $\text{MOD}_{10} : \mathbb{N} \rightarrow \mathbb{N}$ be defined, for every $n \in \mathbb{N}$, by $\text{MOD}_{10}(n)$ is the remainder after the division of $n$ by 10.

   I) Write a pseudocode program that computes $\text{MOD}_{10}$.
   II) Write a GOTO program that computes $\text{MOD}_{10}$.

c) Let $q : \mathbb{N} \rightarrow \mathbb{N}$ be defined, for every $n \in \mathbb{N}$, by $q(n)$ is the sum of the digits in $n$.

   I) Write a pseudocode program that computes $q$.
   II) Write a GOTO program that computes $q$.

**Remark:** You are allowed to use $\text{DIV}_{10}$ and $\text{MOD}_{10}$ as auxiliary programs.