Exercises for Advances in Theoretical Computer Science
Exercise Sheet 5
Due at 29.11.2021, 10:00 s.t.

Remark: For writing pseudocode, use the language that is defined at the home page of the exercise. For register machine programs, you are allowed to use the instructions defined at the home page of the exercise.

Exercise 5.1
Let $P$ be the following GOTO program:

1: $x_4 := x_1$
2: if $x_4 = 0$ goto 10;
3: $x_5 := x_2$
4: if $x_5 = 0$ goto 8;
5: $x_3 := x_3 + 1$
6: $x_5 := x_5 - 1$
7: if $x_6 = 0$ goto 4;
8: $x_4 := x_4 - 1$
9: if $x_6 = 0$ goto 2;
10: $x_5 := x_5 - 1$

a) Which value does $P$ compute on the following inputs:

I) $x_1 = 2$, $x_2 = 0$,  
II) $x_1 = 0$, $x_2 = 3$,  
III) $x_1 = 2$, $x_2 = 3$

b) Which function $f : \mathbb{N}^2 \rightarrow \mathbb{N}$ is computed by $P$?

c) Use the transformation presented in the lecture to construct a WHILE-IF program with the same semantics as $P$. 
Exercise 5.2

Let \( P \) be the following WHILE program:

\[
\begin{align*}
x_4 &:= 10 - x_1; \\
x_5 &:= 1 - x_4; \\
\text{while } x_5 \neq 0 \text{ do} \\
& \quad x_5 := x_5 + 1 \\
\end{align*}
\]

\[
\begin{align*}
x_4 &:= x_2 - 1; \\
x_5 &:= x_1; \\
x_3 &:= x_1; \\
\text{while } x_4 \neq 0 \text{ do} \\
& \quad x_5 := x_5 \times 10; \\
& \quad x_3 := x_3 + x_5; \\
& \quad x_4 := x_4 - 1 \\
\end{align*}
\]

\[
x_5 := 0
\]

a) Which value does \( P \) compute on the following inputs:

I) \( x_1 = 2, x_2 = 3, \)

II) \( x_1 = 3, x_2 = 0 \)

b) Which function \( f : \mathbb{N}^2 \rightarrow \mathbb{N} \) is computed by \( P \)?

c) Use the transformation presented in the lecture to construct a GOTO program which has the same semantics as \( P \).

Exercise 5.3

Let \( P \) be the following WHILE program:

\[
\begin{align*}
x_3 &:= 0; \\
x_4 &:= x_2; \\
\text{while } x_1 \neq 0 \text{ do} \\
& \quad x_2 := x_4; \\
& \quad \text{while } x_2 \neq 0 \text{ do} \\
& \quad \quad x_3 := x_3 + 1; \\
& \quad \quad x_2 := x_2 - 1 \\
& \quad \end{align*}
\]

\[
x_1 := x_1 - 1
\]

Find a WHILE-IF program \( P' \) with one WHILE loop only which has the same semantics as \( P \) (i.e. \( \Delta(P') = \Delta(P) \)). Use for this the results on the slides from 24.11.2021.
Exercise 5.4

Remark: For the following three tasks, you do not need to write precise programs. It is sufficient to explain your approach.

a) Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be a bijective function which is WHILE computable. Describe why its inverse, $f^{-1} : \mathbb{N} \rightarrow \mathbb{N}$, is WHILE computable as well.

b) Is there a GOTO computable bijection $g : \mathbb{N} \rightarrow \mathbb{N}$ for which $g^{-1} : \mathbb{N} \rightarrow \mathbb{N}$ is not GOTO computable? Justify your answer.

c) Let $f : \mathbb{N}^2 \rightarrow \mathbb{N}$ be a bijective function which is WHILE computable. Let $P$ be the WHILE program which computes $f$.

Describe the idea how a WHILE program can be constructed which uses $P$ and behaves as follows: Started with input $n_1$ in register $x_1$ it ends with value $n_2$ in register $x_2$ and value $n_3$ in register $x_3$, where $n_2$ and $n_3$ are such that $f(n_2, n_3) = n_1$. 

If you want to submit solutions, please do so until 29.11.2021, 10:00 s.t. via e-mail (with “Homework ACT CS” in the subject) to dpeuter@uni-koblenz.de.