Exercise 10.1

Let $S$ be a set of states (including the halting state). Are the following problems decidable or undecidable? Justify your answer.

I) $P_1 := \{(n,q) \in \mathbb{N} \times S \mid \text{there exists a start configuration } (s,\#w\#) \text{ of } M_n \text{ from which state } q \text{ is reachable}\}$

II) $P_2 := \{(n,w) \in \mathbb{N} \times \Sigma^* \mid \text{there exists } q \in K \text{ which does not occur in the computation of } M_n = (K,\Sigma,\delta,s) \text{ with start configuration } (s,\#w\#)\}$

**Remark:**

- $M_n$ denotes the Turing machine with Gödel number $n$.

**Hint:** To prove undecidability you can for instance use a reduction to a problem which was already proved to be undecidable in the lecture or a previous exercise.

Exercise 10.2

Prove that it is undecidable whether a WHILE program which computes a partial function $f: \mathbb{N} \rightarrow \mathbb{N}$ terminates on input $n$.

**Hint:** One can give e.g. a proof by contradiction using the fact that the class of WHILE-computable functions coincides with the class of TM-computable functions.

Exercise 10.3

Prove that the following problems are undecidable using the theorem of Rice.

I) $L_1 = \{n \mid M_n \text{ accepts an infinite language} \}$

II) $L_2 = \{n \mid M_n \text{ accepts a finite language} \}$

III) $L_3 = \{n \mid M_n \text{ accepts a decidable language} \}$

IV) Let $k \in \mathbb{N}$ and $L_4 = \{n \mid M_n \text{ accepts only words which have length greater than } k\}$

V) $L_5 = \{n \mid L(M_n) \text{ is context sensitive} \}$

VI) $L_6 = \{n \mid \text{the language accepted by } M_n \text{ is regular} \}$

VII) $L_7 = \{n \mid M_n \text{ halts on all inputs } w \in \Sigma^\ast \}$