Exercise 12.1
Prove that for every alphabet $\Sigma$ with $|\Sigma| \geq 2$ it is undecidable whether for DCFL languages $L_1, L_2$ we have $L_1 \subseteq L_2$.

Hint: Reduction to the problem of testing emptiness for intersection of DFCL languages. Use the fact that the complement of a DCFL language is a DCFL language.

Exercise 12.2

Definition: A map $h : \Sigma_1^* \rightarrow \Sigma_2^*$ is a monoid homomorphism if it has the property that for all words $w_1, w_2 \in \Sigma_1^*$, $h(w_1w_2) = h(w_1)h(w_2)$.

Prove that the following problem is undecidable:
Let $f, g : \Sigma_1^* \rightarrow \Sigma_2^*$ be monoid homomorphisms. Assume that $\Sigma_1 = \{a_1, \ldots, a_n\}$.
Is there a word $w \in \Sigma_1^*$ such that $f(w) = g(w)$?

Hint: Note that $f$ and $g$ are completely described by their values on $a_1, \ldots, a_n$.

Exercise 12.3

Definitions: Assume we are in propositional logic with propositional variables $\Pi$.

- A literal $L$ is a propositional variable $P$ or the negation of a propositional variable $\neg P$.

- A propositional formula is in disjunctive normal form (DNF) if it has the form $(L_1^1 \land \cdots \land L_{n_1}^1) \lor \cdots \lor (L_1^m \land \cdots \land L_{n_m}^m)$.

- A propositional formula is a clause if it is of the form $L_1 \lor \cdots \lor L_n$ (i.e. is a disjunction of literals). A Horn clause is a clause which contains at most one positive literal. (For instance $P \lor \neg Q \lor \neg R$ and $\neg Q \lor \neg R$ are Horn clauses, but $P \lor Q \lor \neg R$ is not.)

a) Is it true that for every formula $F$ in disjunctive normal form we can check whether $F$ is satisfiable in polynomial time? Briefly justify your answer.

b) Is it true that for every formula $F$ which is a conjunction of Horn clauses we can check whether $F$ is satisfiable in polynomial time? Briefly justify your answer.

Remark: For answering these questions you do not need to construct Turing machines. You can use results on propositional logic presented e.g. in the lecture “Logik für Informatiker” (e.g. the existence of algorithms for checking the satisfiability of sets (= conjunctions) of Horn clauses).

If you want to submit solutions, please do so until 24.01.2022, 10:00 s.t. via e-mail (with “Homework ACTCS” in the subject) to dpeuter@uni-koblenz.de