

Online Dating Recommender Systems: The Split-complex Number Approach

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ABSTRACT

A typical recommender setting is based on two kinds of relations: similarity between users (or between objects) and the taste of users towards certain objects. In environments such as online dating websites, these two relations are difficult to separate, as the users can be similar to each other, but also have preferences towards other users, i.e., rate other users. In this paper, we present a novel and unified way to model this duality of the relations by using split-complex numbers, a number system related to the complex numbers that is used in mathematics, physics and other fields. We show that this unified representation is capable of modeling both notions of relations between users in a joint expression and apply it for recommending potential partners. In experiments with the Czech dating website Libimseti.cz we show that our modeling approach leads to an improvement over baseline recommendation methods in this scenario.

Categories and Subject Descriptors

H.4 [Information Systems Applications]: Miscellaneous;
D.2.8 [Software Engineering]: Metrics—*complexity measures, performance measures*

General Terms

Theory, Experimentation, Algorithms

Keywords

Recommender system, online dating, split-complex numbers

1. INTRODUCTION

Recommender systems typically come in two flavors: users receive recommendations for objects or they receive recommendations for other users. The recommendation of objects occurs, for instance, in online shops where a user receives suggestions for other interesting products based on his context. In this case the world of recommendations is asym-

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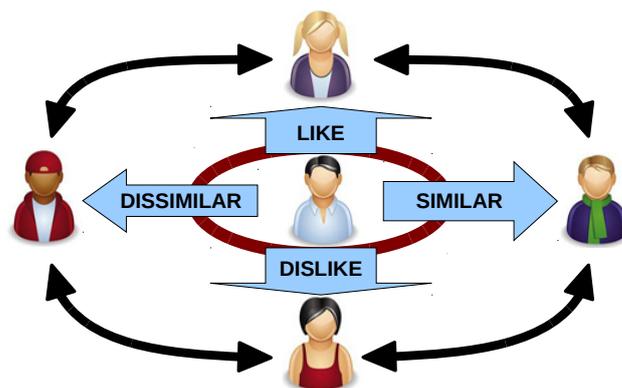


Figure 1: The four kinds of relationships modeled in our approach: Like, dislike, similarity and dissimilarity.

metric and clearly divided into *those who receive a recommendation* (the user) and *that what is recommended* (the product). The recommendation then corresponds to predicting a relation of type *taste* between a subject and an object. The alternative setting of recommending users to a user, instead, is a typical application that can be found in classical social networks. Here, a user receives suggestions for people he might know based on his social context. In this case the setting is symmetric, as there is no distinction between the type of the object that is recommended and the one that receives the recommendation (both are users). If we consider homophily among the users as an important factor, the recommendation in this scenario corresponds to predicting a relation of the type *similar* between two subjects.

There are however settings which lie at the intersection of those two scenarios. Take, for instance, classical dating websites, where users are interested in getting to know other users. At first look, this seems to be a homophily setting and it seems advisable to make recommendations based on the relation of type *similar*. However, in the classical setting the users might be more interested in getting to know people of the respective other gender. Thus, we have an asymmetry in the scenario as we typically would not recommend women to women nor men to men. Hence, we are also dealing with the prediction of relation of type *taste* between different types of users (i.e., men and women).

Figure 1 shows the four kinds of relationships encoun-

tered in dating sites: like, dislike, similarity and dissimilarity. While it would be possible to model these four relationships as two asymmetric recommendation scenarios (recommending men to women and recommending women to men), in this paper we propose a novel and integrated model to address this task based on split-complex numbers. The split-complex numbers are a number system related to the complex numbers. Whereas the complex numbers are defined by introducing a non-real number i with the property $i^2 = -1$, the split-complex are defined by introducing a non-real number j with the property $j^2 = +1$. We show that split-complex numbers provide a natural way to model the particularities of online dating sites. Furthermore, our modeling approach allows for reducing the problem of dating recommendations to a link prediction problem – a problem that has been analyzed well in research on recommender systems.

In the rest of this paper, we proceed as follows: In the next section we give a brief introduction to related work in the field of recommender systems for online dating websites. Then, we describe the typical approach to model this scenario based on real numbers in Section 3, before developing the representation based on split-complex numbers in Section 4. We proceed in Section 5 with a description of the complete algorithm and evaluate the method in Section 6. We conclude the paper in Section 7.

2. BACKGROUND

Dating sites present a special case for recommender systems. While in ordinary recommender systems, an item is recommended to a person, dating sites try to recommend two people to each other, based on the tastes of both users. Thus, dating recommender systems combine the characteristics of social recommender systems, in which users are recommended to each other, but in which friendships are usually undirected, and item recommender systems, in which the rating graph is bipartite. Thus, a dating recommender system must possess two properties:

- It must be reciprocal, i.e., find pairs of users that are both likely to like each other.
- It must distinguish between the like/dislike relationships and the similar/dissimilar relationships.

The first requirement, reciprocity, has been addressed in previous dating recommender systems. The second requirement however, has not yet been considered. To build a dating recommender system that fulfills this second requirement, this paper introduces a model of dating networks based on split-complex networks.

Recommender systems for online dating are related to the more general problem of social matching [19], such as the user model and recommendation systems for matchmaking for private discussions in [1], and a system for recommending workers in a distributed workplace environment [6]. For recommender systems with people on both sides of the recommendation, i.e., the subject and object of a recommendation, bidirectional or reciprocal recommendations match between people according to preferences of both sides. Bilateral recommendations are used for matching between people and jobs [12], following the argumentation that a globally optimal matching needs to consider preferences of both sides.

Other reciprocal recommenders for online dating are proposed in [2] and [16].

In previous work on dating recommender systems, reciprocity has played a dominant role. In short, it is essential that recommendations occur if both people like each other. The recommendation considers personal profile information and follows the observation and study in [5] that matching with respect to similar personal preferences leads to higher matching quality. However, as mentioned in [14] personal profiles lack information and therefore limit recommendation potential, while implicit user preferences might compensate this. As in our work, recommendations take implicit user preferences into account, i.e., preferences that are derived from user’s rating behavior (cf. [15]).

3. USING THE REAL NUMBERS FOR RECOMMENDATION

Our recommender approach is based on the social graph between users, i.e., on ratings of users by other users. This approach is similar to the social recommendation problem, for instance that of recommending friends on Facebook. In fact, our proposed method can be derived from social recommender algorithms by replacing the use of real numbers by the use of split-complex numbers. Therefore, we will first describe social recommenders that use the real numbers in this section, and then generalize them to split-complex numbers in later sections.

3.1 Recommending Friendship

Let us assume that in a social graph, the only possible relationship is that of *friend*. Then, the social recommendation problem consists of recommending new friends based on existing friendships. The main tool used for this purpose is the principle of *triangle closing*: people who have (maybe many) common friends might be friends themselves. Figure 2 illustrates the principle of triangle closing.

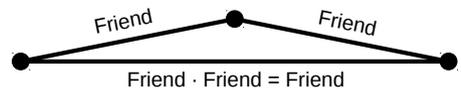


Figure 2: Triangle closing in a network with only the *friend* relationship. Two adjacent *friend* edges let us predict a new *friend* edge.

In this model, there is only one relationship type (friendship), and thus all predicted edges are of this type. If however a network contains multiple relationship types, then a rule must be defined for combining two known edges into a third one. One example for this case are networks which model enemies in addition to friends.

3.2 Recommending Friendship and Enmity

In some social networks such as that of the technology news website Slashdot, people can tag each other as *friends* and *foes* [11]. The resulting social graph contains positive and negative edges. In such a social network, the principle of triangle closing can be generalized to the multiplication rule using $+1$ for friendship and -1 for enmity. The resulting recommender will follow the adage “The enemy of my enemy is my friend”, due to the relation $-1 \cdot -1 = +1$. Figure 3

illustrates the principle of triangle closing in networks with negative edges.

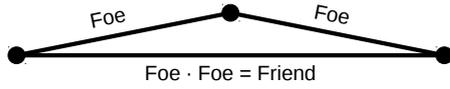


Figure 3: Triangle closing in a network with *friend* and *foe* relationships. In such a network, new edges can be predicted using the multiplication rule that for instance states that “The enemy of my enemy is my friend”.

Accordingly, recommender systems for signed social networks will take into account the multiplication rule of real numbers.

4. OUR MODEL: THE SPLIT-COMPLEX NUMBERS

In this section, we present our model of the profile rating graph, using the split-complex numbers. The split-complex numbers are an extension of the real numbers similar to the complex numbers [17]. Instead of including an imaginary number i such that $i^2 = -1$, the split-complex numbers include an imaginary number j such that $j^2 = +1$. Using split-complex numbers, we can represent the *like* and *dislike* relationships using the numbers $+j$ and $-j$, and the *similar* and *dissimilar* relationships using the real numbers $+1$ and -1 . Figure 4 shows this assignment of these four relations to unit split-complex numbers.

When users rate the profile of other users on a dating site, we get directed edges denoting *like* and *dislike*. Let us imagine two men A and B that like the same woman C. Then, by triangle closing, we may predict an edge between A and B. This new edge however will not denote a *like*. Instead, it will denote similarity. Therefore, we really need two kinds of relationships in this network: *like* and *similar*. Let us represent these two possible values by e_{like} and e_{similar} . Now, remember that in the triangle closing model of Section 3, we multiply the weights of two adjacent edges to generate a new edge. In the case of a dating site, we can formulate the following natural triangle closing rules shown in Figure 5.

These rules can be expressed mathematically in the following way:

$$\begin{aligned} e_{\text{like}} \cdot e_{\text{like}} &= e_{\text{similar}} \\ e_{\text{similar}} \cdot e_{\text{similar}} &= e_{\text{similar}} \\ e_{\text{like}} \cdot e_{\text{similar}} &= e_{\text{like}} \end{aligned}$$

We thus have to find values of e_{like} and e_{similar} that solve these equations, and in which both constants are nonzero. A trivial solution is given by $e_{\text{like}} = e_{\text{similar}} = 1$. However, this trivial solution is not satisfactory, since we want the relationships *like* and *similar* to be different. From the second and third equations, we can derive that $e_{\text{similar}} = 1$. Therefore, e_{like} is a number different from zero and from 1 that squares to 1. Since no real number has these properties, we have to introduce a non-real value for e_{like} such that $e_{\text{like}}^2 = 1$. This construction corresponds to the split-complex

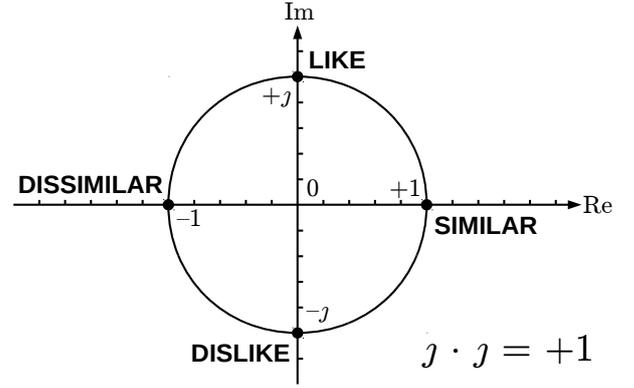


Figure 4: The split-complex number plane used in our model. The four unit split-complex numbers $+1$, -1 , $+j$ and $-j$ are used to represent the four relationship types *similar*, *dissimilar*, *like* and *dislike*.

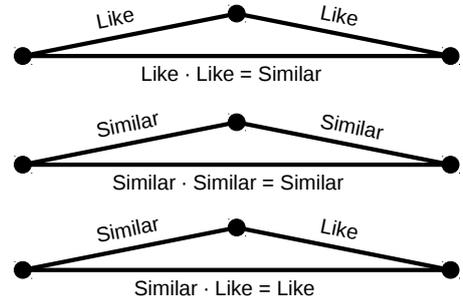


Figure 5: Triangle closing in a dating network. These multiplication rules lead to the assignment of the value $+j$ to the *like* relationship and of the value $+1$ to the *similar* relationship.

numbers, where the imaginary unit j squares to one:

$$\begin{aligned} e_{\text{like}} &= j \\ e_{\text{similar}} &= 1 \end{aligned}$$

Our three requirements then correspond to the identities $j \cdot j = 1$, $1 \cdot 1 = 1$ and $j \cdot 1 = j$ that hold for the split-complex numbers. Analogous multiplication rules with *dislike* and *dissimilar* can then be derived by multiplying both sides with -1 . For instance, the equation $+j \cdot -1 = -j$ states that a *like* combined with a *dissimilar* results in a *dislike*.

4.1 Formal Definition

The split-complex numbers were introduced in 1848 by James Cockle as a special case of the *tessarines* and were thus called *real tessarines* [4]. They can be defined formally as the set $C_s = \{a+bj \mid a, b \in \mathbb{R}\}$, together with the rule $j^2 = +1$. From this, the following addition and multiplication rules can be derived:

$$\begin{aligned} (a + bj) + (c + dj) &= (a + c) + (b + d)j \\ (a + bj)(c + dj) &= (ac + bd) + (ad + bc)j \end{aligned}$$

Note the similarity to the corresponding rules for the complex numbers, which differ only by a single sign change.

Unlike the complex numbers, \mathbb{C}_s is not a field. Instead, \mathbb{C}_s is a commutative ring, i.e., all field axioms are valid except for the existence of the multiplicative inverse, which does not exist for numbers of the form $a \pm aj$. As a result, products of two nonzero numbers can be zero, e.g., $(1+j)(1-j) = 0$. Due to these defects, the split-complex are used much less than complex numbers.

The split-complex numbers are often studied together with number systems such as the quaternions, for instance by William Clifford in 1873 [3]. Split-complex numbers are also called hyperbolic complex numbers because they can represent hyperbolic angles [7]. In the context of special relativity for instance, numbers of the form $a + bj$ with $a^2 - b^2 = 1$ are used to model Lorentz boosts. Other applications of hyperbolic angles are squeeze mappings in geometry, giving them the alternative name *hyperbolic numbers* [18]. Other names are *tessarines*, *countercomplex numbers*, *anormal-complex numbers* and *Lorentz numbers*.

5. PROPOSED ALGORITHM

In the model derived in the previous section, we established that *like* edges should be represented by the split-complex weight $+j$ and *dislike* edges by the split-complex weight $-j$. We will now use this model to derive a dating recommendation algorithm.

5.1 Algebraic Graph Theory

In unweighted networks, it is a well-known result that the number of paths connecting two nodes can be computed using powers of matrices. Let $G = (V, E)$ be an unweighted and undirected network. Its adjacency matrix is then defined as the matrix $\mathbf{A} \in \mathbb{R}^{|V| \times |V|}$ given by

$$\mathbf{A}_{ij} = \begin{cases} 1 & \text{when } \{i, j\} \in E, \\ 0 & \text{when } \{i, j\} \notin E. \end{cases}$$

The adjacency matrix \mathbf{A} is square and symmetric. Given two nodes $i, j \in V$, the number of common neighbors of i and j is given by the square of the adjacency matrix:

$$\text{CN}(i, j) = (\mathbf{A}^2)_{ij}$$

Equivalently, the number of common neighbors of i and j can be interpreted as the number of paths of length two between i and j . This characterization can be generalized to paths of any length: The entry $(\mathbf{A}^k)_{ij}$ equals the number of paths of length k from node i to node j .

Thus, the powers of the adjacency matrix \mathbf{A} can be used to implement a social recommender based on triangle closing:

- \mathbf{A}^2 implements the basic triangle closing recommender, i.e., counting the number of triangles that can be closed.
- Higher powers \mathbf{A}^k generalize triangle closing to the closing of longer paths.

An example of such a social recommendation algorithm combining these results is the matrix exponential:

$$\exp(\mathbf{A}) = \mathbf{I} + \mathbf{A} + \frac{1}{2}\mathbf{A}^2 + \frac{1}{6}\mathbf{A}^3 + \dots$$

This function is suitable as a social recommendation function since it takes all paths between two nodes into account (containing all powers of \mathbf{A}), and because short paths are given preference over long paths (the weights of the powers are decreasing).

We will now generalize this method to networks with multiple relationship types. In networks with *friend* and *foe* links for instance, powers of \mathbf{A} can also be used, as they automatically implement the multiplication rule.

Let $G = (V, E, \sigma)$ be a signed graph, i.e., a graph with positive and negative edges, in which σ is the sign function. The sign function σ is a function from the set of edges E to $\{+1, -1\}$ giving the sign of all edges. The adjacency matrix of G is then defined as

$$\mathbf{A}_{ij} = \begin{cases} +1 & \text{when } \{i, j\} \in E \text{ and } \sigma(\{i, j\}) = +1, \\ -1 & \text{when } \{i, j\} \in E \text{ and } \sigma(\{i, j\}) = -1, \\ 0 & \text{when } \{i, j\} \notin E. \end{cases}$$

The square of \mathbf{A} then gives, for each pair (i, j) , the number of common neighbors of i and j connected by the same edge weight minus the number of common neighbors connected by different edge weights. Thus, $(\mathbf{A}^2)_{ij}$ is a link prediction function that implements the multiplication rule illustrated by the phrase ‘‘The enemy of my enemy is my friend’’.

Analogously to the unsigned case, higher powers \mathbf{A}^k represent a count of all paths of length k between two nodes, weighted positively if the path is positive (i.e., the product of edge weights is positive) and negatively if the path is negative (i.e., the product of edge weights is negative). Thus, the matrix exponential $\exp(\mathbf{A})$ can be used as a recommendation algorithm in networks with positive and negative edges, giving a recommender that takes all paths between two nodes into account, weighted by the sign of the path.

The case of networks with positive and negative edges thus shows that the rule ‘‘The enemy of my enemy is my friend’’ corresponds to the multiplication rule of the real numbers $\{+1, -1\}$, and thus positive and negative edges can be represented by $+1$ and -1 when defining the adjacency matrix of the network. In the next section, we will generalize this technique to dating networks with *like*, *dislike*, *similar*, and *dissimilar* edges.

5.2 Split-complex Path Counting

Let $G = (V, E, w)$ be the directed and signed rating network, in which V is the set of users and E is the set of directed edges. Each edge $(i, j) \in E$ is given a weight by the weight function w , with the weight $w((i, j))$ being any nonzero real number. A positive number denotes a *like* relationship and a negative number denotes a *dislike* relationship. Using these definitions, we will define the real adjacency matrix \mathbf{A} as

$$\mathbf{A}_{ij} = \begin{cases} w((i, j)) & \text{when } (i, j) \in E, \\ 0 & \text{when } (i, j) \notin E. \end{cases}$$

Thus, the split-complex adjacency matrix of the network is $\mathbf{A}_s = j\mathbf{A} \in \mathbb{C}_s^{|V| \times |V|}$.

Based on the multiplication rules of the split-complex numbers described in Section 4, the powers of \mathbf{A}_s implement our model of likes and similarities. The weighted path count for paths of length k is then given by

$$\mathbf{A}_s^k = j^k \mathbf{A}^k,$$

and thus the interpretation of a path of length k depends on whether k is even or odd:

$$j^k = \begin{cases} 1 & \text{when } k \text{ is even} \\ j & \text{when } k \text{ is odd} \end{cases}$$

Thus, any sum of the powers of \mathbf{A}_s can be split into even and odd components, giving a real and imaginary part of the result. Let

$$p(\mathbf{X}) = a\mathbf{I} + b\mathbf{X} + c\mathbf{X}^2 + d\mathbf{X}^4 + \dots$$

be a power sum of the matrix \mathbf{X} . As an example, $p(\mathbf{X})$ could be the matrix exponential. This power sum can be applied to \mathbf{A}_s giving

$$\begin{aligned} p(\mathbf{A}_s) &= a\mathbf{I} + b\mathbf{A} + c\mathbf{A}^2 + d\mathbf{A}^3 + \dots \\ &= (a\mathbf{I} + c\mathbf{A}^2 + \dots) + \mathbf{j}(b\mathbf{A} + d\mathbf{A}^3 + \dots). \end{aligned}$$

We thus see that the even part of the power sum can be used to find similar persons, and the odd part to find liked persons. We see that to predict *like* edges, only paths of odd lengths should be used, and that all paths of even length lead to predictions of similarity instead.

5.3 Representation as 2×2 Matrices

Any split complex number can be represented as a 2×2 matrix in the following way:

$$a + b\mathbf{j} \equiv \begin{bmatrix} a & b \\ b & a \end{bmatrix}.$$

In this representation, the addition and multiplication of split-complex numbers corresponds to the addition and multiplication of 2×2 matrices. The units 1 and \mathbf{j} then correspond to

$$\begin{aligned} 1 &\equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ \mathbf{j} &\equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \end{aligned}$$

Using this representation, the split-complex adjacency matrix \mathbf{A}_s can be represented by the real matrix \mathbf{A}_b :

$$\mathbf{A}_s \equiv \mathbf{A}_b = \begin{bmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{A}^T & \mathbf{0} \end{bmatrix}.$$

By computing the powers of \mathbf{A}_b , we recover the property by which paths of even and odd length are separated:

$$\begin{aligned} \mathbf{A}_b^{2k} &= \begin{bmatrix} (\mathbf{A}\mathbf{A}^T)^k & \mathbf{0} \\ \mathbf{0} & (\mathbf{A}^T\mathbf{A})^k \end{bmatrix} \\ \mathbf{A}_b^{2k+1} &= \begin{bmatrix} \mathbf{0} & (\mathbf{A}\mathbf{A}^T)^k \mathbf{A} \\ (\mathbf{A}^T\mathbf{A})^k \mathbf{A}^T & \mathbf{0} \end{bmatrix} \end{aligned}$$

5.4 Representation as the Bipartite Double Cover

There is another equivalent way to recover the property that paths of odd length are needed for predicting *like* edges. This method is based on the bipartite double cover of the network. The bipartite double cover of a directed graph is an undirected graph with twice the number of nodes and the same number of edges. The construction of the bipartite double cover is illustrated in Figure 6.

In the bipartite double cover, each node i gives two nodes i_{out} and i_{in} , and each directed edge (i, j) gives the undirected edge $\{i_{out}, j_{in}\}$. Thus, the bipartite double cover splits each node into a node that keeps all the outlinks and one that keeps all the inlinks. The resulting graph is bipartite, since every edge connects an *out* node with an *in* node. The bipartite double cover of a graph G is denoted $G \times K_2$ since it

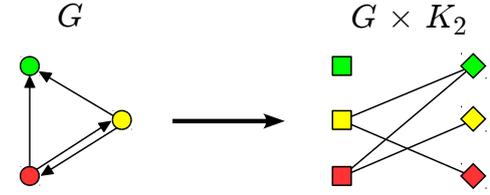


Figure 6: The construction of the bipartite double cover of a directed graph: Each node is separated into two nodes, one which inherits all outlinks, and one which inherits all inlinks.

also corresponds of the tensor product of G with the complete graph on two nodes K_2 .

The equivalence of the bipartite double cover with the split-complex adjacency matrix can be seen by considering the relation between the two resulting adjacency matrices: If the graph G has the real adjacency matrix \mathbf{A} , then its bipartite double cover has the adjacency matrix

$$\mathbf{A}_b = \begin{bmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{A}^T & \mathbf{0} \end{bmatrix}.$$

This is exactly the alternative representation of the split-complex adjacency matrix \mathbf{A}_s , and thus their powers are equivalent. This shows that counting paths in a graph with edge weights $\pm \mathbf{j}$ is equivalent to counting paths in the bipartite double cover of the graph with edge weights ± 1 .

5.5 Reduction to the Singular Value Decomposition

In the previous section, we showed that computing powers of the split-complex adjacency matrix \mathbf{A}_s is equivalent to computing powers of the bipartite adjacency matrix $\mathbf{A}_b = [\mathbf{0}\mathbf{A}; \mathbf{A}^T\mathbf{0}]$. To compute powers or power sums of a symmetric matrix, we can use its eigenvalue decomposition in the way described in this section.

The eigenvalue decomposition of a real symmetric matrix \mathbf{M} is given by an orthogonal matrix \mathbf{U} and a diagonal $\mathbf{\Lambda}$ such that

$$\mathbf{M} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T.$$

The singular value decomposition of any rectangular or square real matrix \mathbf{M} is given by orthogonal matrices \mathbf{U} and \mathbf{V} and a diagonal matrix $\mathbf{\Sigma}$ of the same size as \mathbf{M} such that

$$\mathbf{M} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T.$$

Now, since the matrix \mathbf{A}_b has the structure $\mathbf{A}_b = [\mathbf{0}\mathbf{A}; \mathbf{A}^T\mathbf{0}]$ its eigenvalue decomposition can be reduced to the singular value decomposition of \mathbf{A} [13]. Given the singular value decomposition $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$, the eigenvalue decomposition of \mathbf{A}_b is given by

$$\mathbf{A}_b = \begin{bmatrix} \bar{\mathbf{U}} & \bar{\mathbf{U}} \\ \bar{\mathbf{V}} & -\bar{\mathbf{V}} \end{bmatrix} \begin{bmatrix} +\mathbf{\Sigma} & \mathbf{0} \\ \mathbf{0} & -\mathbf{\Sigma} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{U}} & \bar{\mathbf{U}} \\ \bar{\mathbf{V}} & -\bar{\mathbf{V}} \end{bmatrix}^T$$

with $\bar{\mathbf{U}} = \mathbf{U}/\sqrt{2}$ and $\bar{\mathbf{V}} = \mathbf{V}/\sqrt{2}$. In this decomposition, each singular value σ corresponds to the eigenvalue pair $\{\pm\sigma\}$. This form of the eigenvalue decomposition of \mathbf{A}_b can now be exploited to define dating recommender functions.

5.6 Dating Recommender Functions

It follows that to compute the top-right component (corresponding to the *like* part) of a power of \mathbf{A}_b , it is sufficient to compute $\mathbf{U}\Sigma\mathbf{V}^T$. All even powers result in a top-right component of $\mathbf{0}$. Thus, we can apply any recommendation algorithm such as the matrix exponential to the dating scenario by keeping only the odd powers of the power sum. In the case of the matrix exponential, this leads to the matrix hyperbolic sine:

$$\sinh(\mathbf{A}) = \mathbf{A} + \frac{1}{6}\mathbf{A}^3 + \frac{1}{120}\mathbf{A}^5 + \dots$$

We can now use this method to derive the following dating recommendation algorithms.

Polynomials.

Any polynomial with only odd powers and nonnegative weights can be used as a dating recommendation algorithm:

$$p(\mathbf{A}) = a\mathbf{A} + b\mathbf{A}^3 + c\mathbf{A}^5 + \dots$$

Rank Reduction.

Rank reduction consists in finding a matrix with maximal rank r which is nearest to the given adjacency matrix. In the resulting matrix, entries of zero which denote unconnected node pairs are assigned nonzero values, which can be used as a recommendation score. The reduction to rank r of the matrix \mathbf{A} can be computed from the singular value decomposition of \mathbf{A} by keeping the r largest singular values and changing all other singular values to zero. Since the singular value decomposition \mathbf{A} is related to the eigenvalue decomposition of \mathbf{A}_b , it follows that the best rank- r approximation to \mathbf{A}_b is given by the truncation of Σ .

Hyperbolic sine.

As shown as an example above, the odd component of the matrix exponential gives the matrix hyperbolic sine:

$$\sinh(\mathbf{A}) = \mathbf{A} + \frac{1}{6}\mathbf{A}^3 + \frac{1}{120}\mathbf{A}^5 + \dots$$

Newman kernel.

The Newman kernel is given by the geometric (or Newman) series

$$(\mathbf{I} - \alpha\mathbf{A})^{-1} = \mathbf{I} + \alpha\mathbf{A} + \alpha^2\mathbf{A}^2 + \alpha^3\mathbf{A}^3 + \dots$$

in which the constant α must be smaller than the inverse of the largest singular value of \mathbf{A} . The restriction of the Newman kernel to only odd powers results in the odd Newman kernel

$$\alpha\mathbf{A}(\mathbf{I} - \alpha^2\mathbf{A}^2)^{-1} = \alpha\mathbf{A} + \alpha^3\mathbf{A}^3 + \alpha^5\mathbf{A}^5 + \dots$$

Due to the equivalence with the bipartite case, these odd kernels were previously used for link prediction in bipartite networks [8].

6. EVALUATION

We perform an evaluation of the proposed model on an online dating dataset from the Czech dating site Libimseti.cz.

We will perform two experiments, as follows: When evaluating a dating recommender system, a straightforward optimization can always be done by considering the gender of the

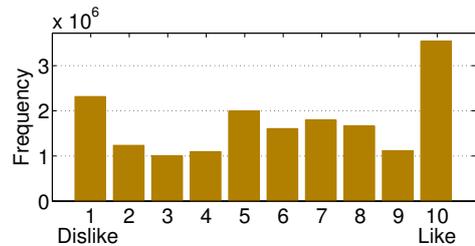


Figure 7: The distribution of rating values in the Libimseti.cz dataset. Ratings in Libimseti.cz are on a scale from 1 (dislike) to 10 (like).

people involved. In the Libimseti.cz dataset, the gender is only known for a subset of all people. Thus, a recommender for that site must be able to work without using gender information. For completeness, we will therefore perform two experiments: The first consists of predicting likes and dislikes without taking into account the gender of people. In the second experiment, we show that likes and dislikes can be predicted also when the gender is known.

6.1 The Libimseti.cz Dataset

Libimseti.cz (from Czech *Libím se ti*, “Do you like me”) is one of the largest Czech dating websites. Our dataset of Libimseti.cz is a unipartite, directed network of users. Edges in the network represent ratings on a scale from 1 (dislike) to 10 (like) of a user by another user. As shown in Table 1, the gender of most users is not known. Figure 7 shows the distribution of ratings. The dataset is available online for download¹.

Since ratings are on a 10-point scale, we must first convert them to positive and negative values. We will achieve this by subtracting from every rating the overall mean rating μ . For each connected pair of users (i, j) , let $r_{ij} \in \{1, \dots, 10\}$ be the rating given on a 10-point scale. Let

$$\mu = |E|^{-1} \sum_{(i,j) \in E} r_{ij}$$

be the overall mean rating. Then we define the real adjacency matrix \mathbf{A} of the network as

$$\mathbf{A}_{ij} = \begin{cases} r_{ij} - \mu & \text{when } (i, j) \in E, \\ 0 & \text{when } (i, j) \notin E. \end{cases}$$

6.2 Experiment: Recommendation with Unknown Gender

The first experiment uses the full Libimseti.cz rating network, in which we ignore the genders of users. The task we evaluate is that of predicting *like* and *dislike* edges. Note that in an actual dating recommender system, two persons can only be matched if they are both predicted to like each other. This is the requirement of reciprocity. However, this requirement can be reduced to predicting a *like* in both directions between the two users, and therefore any like/dislike prediction algorithm can be used to implement a dating recommender. For this reason, we evaluate only the task of like/dislike prediction.

We choose a random set containing 25% of all edges as the test set. The rest of the network is used as the training set.

¹konect.uni-koblenz.de/networks/libimseti

Table 1: Demographics and distribution of ratings in Libimseti.cz.

Gender	Count	Unknown	Rating counts		
			Male	Female	Total
Unknown	83,164	366,180	891,550	445,115	1,702,845
Male	76,441	937,684	682,710	3,232,064	4,852,458
Female	61,365	2,460,765	7,099,688	1,243,590	10,804,043
All	220,970	3,764,629	8,673,948	4,920,769	17,359,346

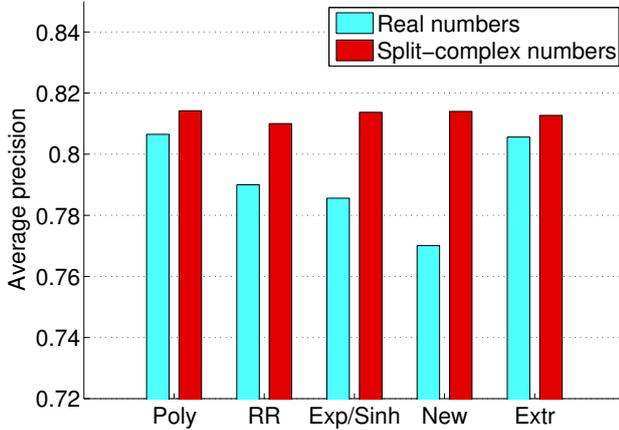


Figure 8: The average precision of all recommender algorithms on the task of predicting likes and dislikes when the gender of users is unknown. Points on the X axis represent different spectral transformations (i.e., graph kernels). The Y axis represents the average precision.

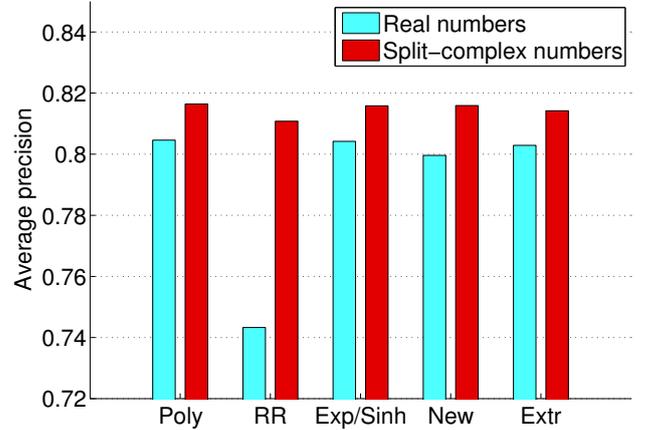


Figure 9: The average precision of all recommender algorithms on the task of predicting likes and dislikes when the gender of users is known. Points on the X axis represent different spectral transformations (i.e., graph kernels). The Y axis represents the average precision.

In the resulting training set, we find the largest connected component, and then keep only the users of that largest connected component in both training and test set. This is to avoid the situation that an edge in the test set would connect two unconnected edges in the training set, which would be effectively unpredictable for all algorithms.

Using this setup, we compute the following algorithms for recommendation:

- (Poly) The best nonnegative polynomial
- (RR) Rank reduction
- (Exp/Sinh) The matrix exponential and hyperbolic sine
- (New) The (odd) Newman kernel
- (Extr) Spectral extrapolation

All methods are computed to a fixed rank of $r = 30$. The first four recommenders are computed using the method described in [10], and the last recommender using the method from [9]. All methods are computed once for the real adjacency matrix and once for the split-complex adjacency matrix (by proxy of the singular value decomposition of \mathbf{A}).

The prediction accuracy is computed as the average precision by comparing the prediction scores from each recommendation algorithm to the actual like/dislike edge weights. The results of the experiment are shown in Fig. 8.

6.3 Experiment: Recommendation with Known Gender

In this experiment, we use the same methodology as in the previous experiment, but restrict the dataset to users with a known gender, and to ratings between users of different genders. Results are shown in Figure 9.

6.4 Discussion

We make the following observations:

- For all recommender systems, our model based on the split-complex numbers performs better than the model using only real numbers. This validates our model that distinguishes the relationship types *like*, *dislike*, *similar* and *dissimilar*.
- The description of our method has been restricted to the case of heterosexual relationships, i.e., men liking women and women liking men. However, the method can in fact be generalized to homosexual relationships: Our first experiment in which we ignored the gender of the rater showed that the method is independent of genders, and thus can be generalized to any types of sexual relationships.

7. CONCLUSION

In this paper, we developed an integrated model to represent both types of relations that can occur among users

in a recommender setting: *similarity* and *taste*. We showed that split-complex numbers provide a natural way to describe how users with the comparable taste are similar to each other and that similar users share the same taste. We conducted thorough experiments on a dataset obtained from an online dating website and showed an improvement in recommendations over the baseline approach.

As directions for future work, we can identify the usage of other number systems for link prediction, such as the complex numbers or the quaternions. In fact, we may state the link prediction problem in a more general way by first laying down requirements such as “A like and a dissimilarity gives a dislike”, and then investigate number systems that fulfill these identities. We hope that this study can serve as an example for how to achieve this.

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