Exercise 3.1: (2 P)
Assume \( P \succ Q \succ R \). Let \( N \) be the following set of clauses:

1. \( \neg R \lor P \)
2. \( \neg Q \lor \neg Q \lor \neg P \)
3. \( Q \)
4. \( R \lor P \)

Let \( S \) be the selection function which selects \( \neg R \) in clause (1) and both occurrences of \( \neg Q \) in clause (2).

Use the ordered resolution calculus with selection \( \text{Res}^S \) described in the lecture for checking the satisfiability of the set \( N \) of clauses.

Exercise 3.2: (2 P)
A propositional Horn clause is a clause which has at most one positive literal.

(Example: \( \neg P \lor Q \lor \neg R \), \( \neg P \lor \neg R \) and \( Q \) are Horn clauses,
whereas \( \neg P \lor Q \lor R \) and \( Q \lor R \) are not Horn clauses.)

Prove: Every set \( H \) of clauses with the following properties:

(i) \( H \) consists only of Horn clauses;
(ii) Every clause in \( H \) contains at least one negative literal;

is satisfiable.

Exercise 3.3: (5 P)
Let \( H \) be a set of propositional Horn clauses. The size of \( H \) is the number of all literals which occur in \( H \). Prove that the resolution calculus \( \text{Res}^S \) (for a suitable selection function \( S \)) can check the satisfiability of \( H \) in time polynomial in the size of \( H \).

Hint: With which choice of the selection function can one model the marking algorithm discussed in the lecture “Logik für Informatiker”?

Supplementary question (will be discussed during the exercise session):
Can you give an algorithm for check the satisfiability of \( H \) in time linear in the size of \( H \)?
Exercise 3.4: (2 P)

Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

1. \((P \lor \neg Q) \land (\neg P \lor Q) \land (Q \lor \neg R) \land (\neg Q \lor \neg R)\)
2. \((P \lor Q \lor \neg R) \land (P \lor \neg Q) \land (P \lor Q \lor R) \land (R \lor Q) \land (R \lor \neg Q) \land (\neg P \lor \neg R) \land \neg U\)

Please submit your solution until Tuesday, November 23, 2021 at 17:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- Use the Homework 03 folder in OLAT (preferred);
- By e-mail to sofronie@uni-koblenz.de with the keyword “Homework DP” in the subject.