Exercises for “Decision Procedures for Verification”

Exercise sheet 7

Exercise 7.1: (3 P)
Let \( \succ \) be a total and well-founded ordering on ground atoms such that, if the atom \( A \) contains more symbols than \( B \), then \( A \succ B \). Let \( N \) be the following set of clauses:

\[
\begin{align*}
\neg q(z, z) \\
\neg q(f(x), y) \lor q(f(f(x)), y) \lor p(x) \\
\neg p(a) \lor \neg p(f(a)) \lor q(f(a), f(f(a))) \\
p(f(x)) \lor p(g(y)) \\
\neg p(a) \lor \neg p(f(a))
\end{align*}
\]

(a) Which literals are maximal in the clauses of \( N \)?
(b) Define a selection function \( S \) such that \( N \) is saturated under \( \text{Res}^{\succ_S} \). Justify your choice.

Exercise 7.2: (3 P)
Let \( \Sigma = (\{f/1, h/1\}, \{p/2, q/2, r/1\}) \). Let \( X \) be a set of variables, and \( \{x, y\} \subseteq X \).

Let \( \succ \) an ordering on ground atoms with the property that for all ground terms \( t_1, \ldots, t_{12} \),

\[
\neg p(t_1, t_2, t_3) \succ p(t_4, t_5, t_6) \succ \neg q(t_7, t_8) \succ q(t_9, t_{10}) \succ \neg r(t_{11}) \succ r(t_{12}).
\]

Let \( N \) be the following set of clauses:

\[
\begin{align*}
\text{(1)} & \quad r(h(x)) \lor r(y) \\
\text{(2)} & \quad \neg q(f(x), y) \lor p(x, x) \\
\text{(3)} & \quad \neg r(h(f(x))) \lor \neg p(x, y) \\
\text{(4)} & \quad q(y, x) \lor p(y, x)
\end{align*}
\]

Use the ordered resolution calculus \( \text{Res}^{\succ} \) described in the lecture for checking the satisfiability of the set \( N \) of clauses.

Exercise 7.3: (2 P)
Let \( F \) and \( G \) be propositional formulae over \( \Pi = \{P, Q, R, S, T, U\} \) such that:

- The CNF of \( F \) is the following set \( N \) of clauses:

\[
\begin{align*}
\text{(1)} & \quad P \lor Q \\
\text{(2)} & \quad \neg P \lor R \lor S \\
\text{(3)} & \quad \neg P \lor \neg R \\
\text{(4)} & \quad P \lor U
\end{align*}
\]
• The CNF of \( \neg G \) consists of the set \( M \) of clauses:

\[
\begin{align*}
5 & : R \lor \neg S \\
6 & : \neg R \lor Q \\
7 & : \neg Q \lor R \\
8 & : \neg S \lor T \\
9 & : S \lor \neg T \\
10 & : \neg Q \lor \neg R
\end{align*}
\]

Which propositional variables occur only in \( N \) and not in \( M \)?

Which propositional variables occur both in \( N \) and in \( M \)?

Use the method described in the lecture to construct a Craig interpolant for \( F \models G \).

**Exercise 7.4:** (5 P)

Assume \( S \succ P \succ Q \succ R \). Let \( N \) be the following set of clauses:

\[
\begin{align*}
1 & : \neg Q \lor P \lor R \\
2 & : \neg R \lor P \\
3 & : Q \lor S \lor \neg P \\
4 & : \neg Q \lor \neg S
\end{align*}
\]

Give the definition of redundancy of a clause w.r.t. a set of clauses.

Is the clause \( \neg Q \lor P \lor S \) redundant w.r.t. the set \( N \) above?

Is the clause \( \neg Q \lor P \) redundant w.r.t. the set \( N \) above? Justify your answer.

Assume \( U \succ S \succ P \succ Q \succ R \). Let \( N \) be the following set of clauses:

\[
\begin{align*}
1 & : \neg Q \lor P \lor R \\
2 & : \neg R \lor P \\
3 & : \neg Q \lor P \lor S \\
4 & : Q \lor S \lor \neg P \\
5 & : \neg Q \lor \neg S
\end{align*}
\]

Is the clause \( \neg Q \lor P \lor S \lor U \) redundant w.r.t. the set consisting of the clauses (1), (2), (4) and (5)? Justify your answer.

**Supplementary exercise**

**Exercise 7.5:** (2 P)

Redundant clauses remain redundant, if the theorem prover deletes redundant clauses. Prove:

If \( N \) and \( M \) are sets of clauses and \( M \subseteq \text{Red}(N) \), then \( \text{Red}(N) \subseteq \text{Red}(N \setminus M) \).

Please submit your solution until Tuesday, December 21, 2021 at 12:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

• Use the folder Homework 07 in OLAT;

• By e-mail to sofranie@uni-koblenz.de with the keyword “Homework DP” in the subject.