Formal Specification and Verification

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Some of the slides are based on or inspired by material by
Wolfgang Ahrendt, Bernhard Beckert, Reiner Hähnle, Andreas Podelski

Motivation

Small faults in technical systems can have catastrophic consequences

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- Operating systems
- Ariane 5
- Mars Climate Orbiter, Mars Sojourner
- Electricity Networks
- Health/devices
- Banks
- Airplanes
- ...

Motivation

Software these days is inside just about anything:

- Cars, Planes, Trains
- Smart cards
- Mobile phones

Software defects can cause failures everywhere

Motivation

Complexity of systems makes verification difficult

- Computer hardware change of scale

  In the 25 last years, computer hardware has seen its performances multiplied by 10^4 to $10^6/10^9$:

  - ENIAC (5000 FLOPS) "Floating-Point Operations per Second"
  - Intel/Sandia Teraflops System (10^{12} FLOPS)

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- The size of the programs executed by these computers has grown up in similar proportions

Achieving Reliability in Engineering

Some well-known strategies from civil engineering

- Precise calculations/estimations of forces, stress, etc.
- Hardware redundancy ("make it a bit stronger than necessary")
- Robust design (single fault not catastrophic)
- Clear separation of subsystems
- Design follows patterns that are proven to work

Why This Does Not Work For Software

- Single bit-flip may change behaviour completely
- Redundancy as replication does not help against bugs
  Redundant SW development only viable in extreme cases
- No clear separation of subsystems
  Local failures often affect whole system
- Software designs have very high logic complexity
- Most SW engineers untrained to address correctness
- Cost efficiency favoured over reliability
- Design practice for reliable software in immature state for complex, particularly, distributed systems

How to Ensure Software Correctness/Compliance?

Testing/Simulation

Testing against inherent SW errors ("bugs")

- design test configurations that hopefully are representative and
- ensure that the system behaves on them as intended

Testing against external faults

- Inject faults (memory, communication) by simulation

Limitations of Testing

- Testing shows the presence of errors, in general not their absence (exhaustive testing viable only for trivial systems)
- Choice of test cases/injected faults: subjective
- How to test for the unexpected? Rare cases?
- Testing is labor intensive, hence expensive

Formal Methods

- Rigorous methods used in system design and development
- Mathematics and symbolic logic
  - precise language / reliable correctness proofs
- Increase confidence in a system

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Make formal model of:

- System implementation
- System requirements

Prove mechanically that formal execution model satisfies formal requirements

Specification

Properties of a system

- Simple properties
  - Safety properties

    "Nothing bad will happen"
  - Liveness properties

    "Something good will eventually happen"

Specification

Properties of a system

- General properties of concurrent/distributed systems
  - deadlock-freedom, no starvation, fairness

Specification

Properties of a system

- Full behavioral specification
  - Code satisfies a contract that describes its functionality
  - Data consistency, system invariants
    (in particular for efficient, i.e. redundant, data representations)
  - Modularity, encapsulation
  - Program equivalence
  - Refinement relation

Formal Methods

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- To replace good design practices

One cannot formally verify messy code with unclear specifications

Correctness guarantees - only for clear requirements and good design

Formal Methods

- Formal proof can replace (infinitely) many test cases
- Formal methods can be used in automatic test case generation
- Formal methods improve the quality of specifications/programs (even without formal verification):
  - better written software (modularization, information hiding)
  - better and more precise understanding of model/implementation
- Formal methods guarantee specific properties of a specific system model

Formal Methods

Problems:

- Formalisation of system requirements is hard

  Oversimplification when modeling
  - 0 delays
  - missing requirements
  - wrong modeling

    (e.g. in the case of programs: $\mathbb{R}$ vs. FLOAT, $\mathbb{Z}$ vz int)

Formal Methods

Problems:

- Proving properties of systems can be hard

Level of System Description

Abstract level

- Finitely many states (finite datatypes)
- Tedious to program, worse to maintain
- Over-simplification, unfaithful modeling inevitable
- Automatic proofs are (in principle) possible

Concrete level

- Infinite datatypes (pointer chains, dynamic arrays, streams)
- Complex datatypes and control structures, general programs
- Realistic programming model (e.g., Java)
- Automatic proofs (in general) impossible;
  positive results in special cases; active area of research

Expressiveness of Specification

Simple

- Simple or general properties
- Finitely many case distinctions
- Approximation, low precision
- Automatic proofs are (in principle) possible

Complex

- Full behavioural specification
- Quantification over infinite domains
- High precision, tight modeling
- Automatic proofs (in general) impossible! positive results in special cases; active area of research

Main approaches

- Concrete programs/Complex properties
- Concrete programs/Simple properties
- Abstract programs/Complex properties
- Abstract programs/Simple properties

Limitations of Formal Methods

Possible reasons for errors:

- Program is not correct (does not satisfy the specification)
  Formal verification proved absence of this kind of error
- Program is not adequate (error in specification)
  Formal specification/verification avoid or find this kind of error
- Error in operating system, compiler, hardware
  Not avoided (unless compiler. operating system, hardware specified/verified)

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In general it is not feasible to fully specify and verify large software systems. Then formal methods are restricted to:

- Important parts/modules
- Important properties/requirements

History

Some of the most important moments in the history of program verification:

History

The idea of proving the correctness of a program in a mathematical sense dates back to the early days of computer science with John von Neumann and Alan Turing.

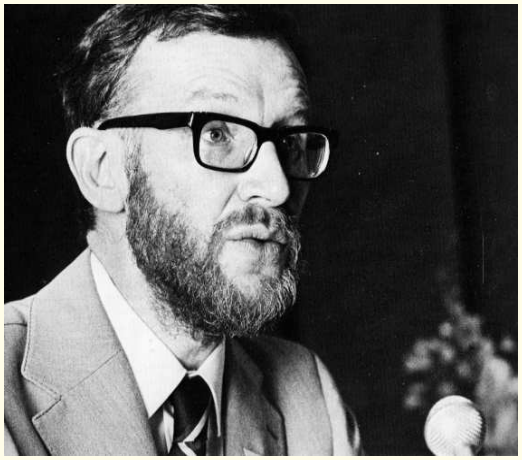
John von Neumann

Alan Turing

History

- R. Floyd and P. Naur introduced the "partial correctness" specification togetherwith the "invariance proof method"
- R. Floyd also introduced the "variant proof method" to prove program termination

Robert Floyd



Peter Naur

History

- C.A.R. Hoare formalized the Floyd/Naur partial correctness proof method in a logic (so-called "Hoare logic") using an Hilbert style inference system;
- Z. Manna and A. Pnueli extended the logic to "total correctness" (i.e. partial correctness + termination).

C.A.R. Hoare

Z. Manna

A. Pnueli

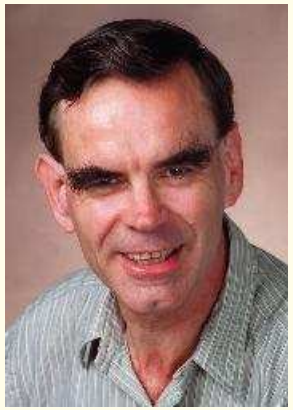
History

Edsger W. Dijkstra introduced predicate transformers (weakest liberal precondition, weakest precondition) and defined a predicate transformer calculus.

Edsger W. Dijkstra

History

Dynamic logic was developed by Vaughan Pratt in 1974 (in notes for a class on program verification) as an approach to assigning meaning to Hoare logic by expressing the Hoare formula $p\{a\}q$ as $p \rightarrow [a]q$. The approach was later published in 1976 as a logical system in its own right.



Vaughan Pratt

The system parallels Edsger Dijkstra's notion of weakest-precondition predicate transformer $wp(a, p)$, with $[a]p$ corresponding to Dijkstra's $wlp(a, p)$, weakest liberal precondition.

History

First attempts towards automation

- James C. King, a student of Robert Floyd, produced the first automated proof system for numerical programs, in 1969.
- The use of automated theorem proving in the verification of symbolic programs (à la LISP) was pionneered, a.o., by Robert S. Boyer and J. Strother Moore

History

Nowadays many theorem provers, many of which are being used for verification: ACL2, COQ, Simplify, SPIN, Key

Model checkers: BLAST, ...

SMT solvers used for verification (Z3, Yices, CVC, ...)

Course Structure

- **Introduction**
- **Specification**
  - Logic (propositional logic, first-order logic)
  - Algebraic specification
  - Transition systems/Abstract-state-machines/Timed automata/Hybrid automata
- **Verification**
  - Temporal logic; Model Checking
  - Basics of deductive verification
    - Hoare Logic and Dynamic Logic
    - Decision procedures for data types.
  - Verification by Abstraction/Refinement (if sufficient time)

Logic

Formal logic:

- Syntax: a formal language (formula expressing facts)
- Semantics: to define the meaning of the language, that is which facts are valid)
- Deductive system: made of axioms and inference rules to formaly derive theorems, that is facts that are provable

- Propositional logic (new: BDD/OBDD)
- First-order logic
- Decidability/undecidability results

Formal specification

- **Specification languages for describing programs/processes/systems**

  Model based specification

  transition systems, abstract state machines, specifications based on set theory

  Axiom-based specification

  algebraic specification

  Declarative specifications

  logic based languages (Prolog)

  functional languages, λ-calculus (Scheme, Haskell, OCaml, ...)

  rewriting systems (very close to algebraic specification): ELAN, SPIKE, ...

- **Specification languages for properties of programs/processes/systems**

  Temporal logic

Algebraic specification

- appropriate for specifying the interface of a module or class
- enables verification of implementation w.r.t. specification
- for every ADT operation: argument and result types (sorts)
- semantic equations over operations (axioms) e.g. for every combination of "defined function" (e.g. top, pop) and constructor with the corresponding sort (e.g. push, empty)
- problem: consistency?, completeness?

Example: Algebraic specification

```
fmod NATSTACK is
  sorts Stack .
  protecting NAT .
  op empty : -> Stack .
  op push : Nat Stack -> Stack .
  op pop : Stack -> Stack .
  op top : Stack -> Nat .
  op length : Stack -> Nat .
  var S S2 : Stack .
  var X Y : Element .
  eq pop(push(X,S)) = S .
  eq top(push(X,S)) = X .
  eq length(empty) = 0 .
  eq length(push(X,S)) =
                1 + length(S) .
endfm
```

Example: Algebraic specification

reduce pop(push(X,S)) == S .

reduce top(pop(push(X,push(Y,S)))) == Y .

reduce S == push(X,S2) implies push(top(S),pop(S)) == S .

reduce S == push(X,S2) implies length(pop(S)) + 1 == length(S) .

- the equations can be used as term rewriting rules
- this allows proving properties of the specification

Transition systems

- model to describe the behaviour of systems
- digraphs where nodes represent states, and edges model transitions
- state:
  - the current colour of a traffic light
  - the current values of all program variables + the program counter
  - the current value of the registers together with the values of the input bits
- transition: ("state change")
  - a switch from one colour to another
  - the execution of a program statement
  - the change of the registers and output bits for a new input

Generalizations of transition systems

- More detailed description of states: Abstract state machines
- Emphasis on processes and their interdependency: CSP

- Durations: Timed automata
- Continuous evolution + discrete control: Hybrid automata

Temporal logic

The purpose of temporal logic (TL) is:

- reasoning about time (in philosophy), and
- reasoning about the behaviour of systems evolving over time (in computer science).

Special language for doing so:

Logical connectives $\wedge, \vee, \neg$

temporal operators: $\bigcirc F$ and $F \mathcal{U} G$

Model Checking

In computer science, model checking refers to the following problem:

> Given a model of a system, test automatically whether this model meets a given specification.

Typically, the systems one has in mind are hardware or software systems, and the specification contains safety requirements such as the absence of deadlocks and/or critical states that can cause the system to crash (which can be expressed in temporal logic).

Model checking is a technique for automatically verifying correctness properties of finite-state systems.

Deductive verification

- **Model checking:**

  Finite transition systems / CTL properties

  States are "entities" (no precise description, except for labelling functions)

  No precise description of actions (only $\rightarrow$ important)

Extensions in two possible directions:

- More precise description of the actions/events
  - Hoare logic
  - Propositional Dynamic Logic
- More precise description of states (and possibly also of actions)
  - succinct representation: formulae represent a set of states
  - deductive verification

Hoare Logic

Hoare logic (also known as Floyd-Hoare logic) is a formal system with a set of logical rules for reasoning rigorously about the correctness of computer programs. It was proposed in 1969 by C. A. R. Hoare.

Central feature: Hoare triple.

A triple describes how the execution of a piece of code changes the state of the computation. A Hoare triple is of the form

$$\{P\}\ C\ \{Q\}$$

where P and Q are assertions and C is a command.

P is named the precondition and Q the postcondition: when the precondition is met, the command establishes the postcondition.

Assertions are formulae in predicate logic.

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Central feature: Hoare triple $\{P\}\ C\ \{Q\}$ (P precondition/Q postcondition)

Hoare logic provides axioms and inference rules for all the constructs of a simple imperative programming language.

Standard Hoare logic proves only partial correctness; termination needs to be proved separately.

> Intuitive reading of a Hoare triple:
> Whenever P holds of the state before the execution of C, then Q will hold afterwards, or C does not terminate.

Dynamic logic

Dynamic logic of programs

Dynamic logic is an extension of modal logic originally intended for reasoning about computer programs and later applied to more general complex behaviors arising in linguistics, philosophy, AI, and other fields.

Operators:
$[\alpha]A$: A holds after every run of the (non-deterministic) process α

$<\alpha>A$: A holds after some run of the (non-deterministic) process α

Dynamic logic permits compound actions built up from smaller actions

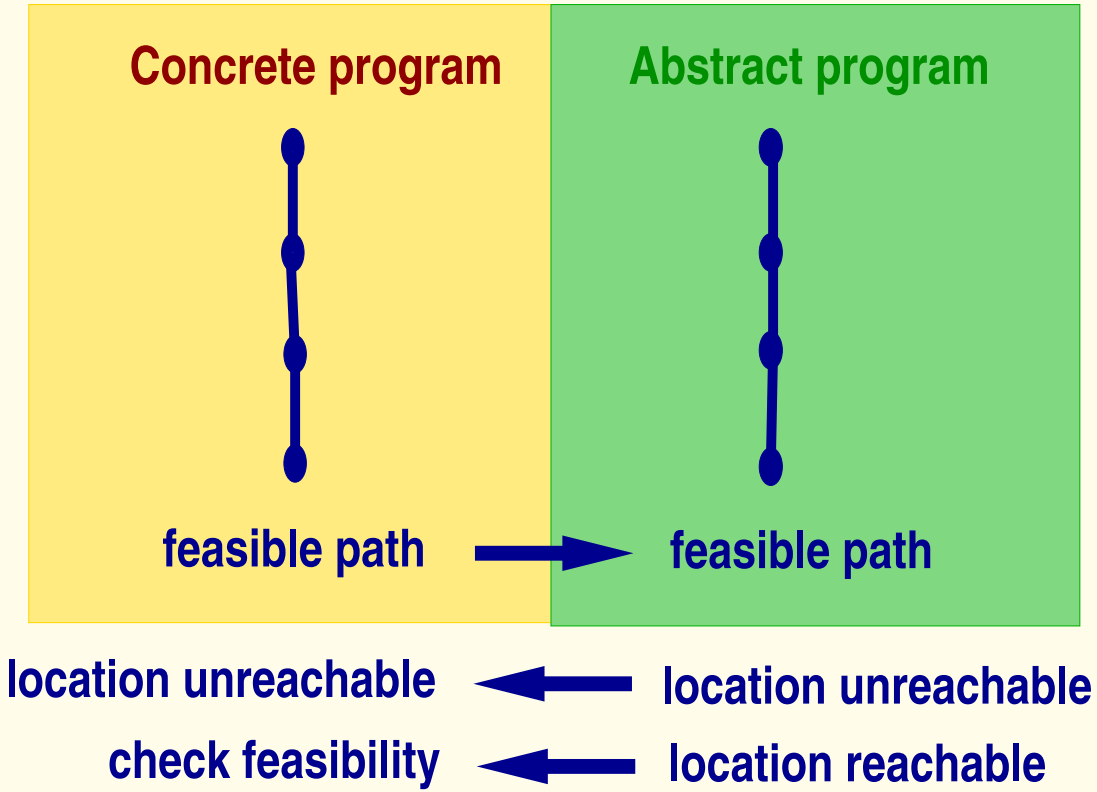
- $\alpha \cup \beta$
- $\alpha; \beta$
- α^*

Decision Procedures

Example: Does BubbleSort return a sorted array?

```
int [] BubbleSort(int[] a) {
    int i, j, t;
    for (i := |a| − 1; i > 0; i := i − 1) {
        for (j := 0; j < i; j := j + 1) {
            if (a[j] > a[j + 1]){t := a[j];
                                 a[j] := a[j + 1];
                                 a[j + 1] := t};
}} return a}
```

Abstraction/Refinement



Organisational Info

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Organisational Info

Course Home Page

www.uni-koblenz.de/∼sofronie/lecture-formal-specif-verif-ss-2016/

Will contain all the information about the course:

- slides
- exercises
- additional information

Passing Criteria

- Written or oral exam (depending on the number of participants)