Exercise 6.1:
You may recall the puzzle of a ferryman, goat, cabbage, and wolf all on one side of a river. The ferryman can cross the river with at most one passenger in his boat. There is a behaviourally
conflict between:

1. the goat and the cabbage; and
2. the goat and the wolf;

if they are on the same river bank but the ferryman is not on that river bank (the goat eats the cabbage, resp. the wolf eats the goat).

Define a “program graph” describing this system: \((\text{Loc}, \text{Act}, \text{Effect}, \rightarrow, \text{Loc}_0)\) where:

- \(\text{Loc} = \{ \text{left}, \text{right}, \text{conflict} \}\) is a set of locations with initial locations \(\text{Loc}_0 = \{ \text{left} \}\).
  - Intuitively, left and right represent the location of the ferryman; conflict represents the conflict situation when the cabbage or the goat is eaten.
- \(\text{Act} = \{ \text{carry-lr-goat}, \text{carry-rl-goat}, \text{carry-lr-cabbage}, \text{carry-rl-cabbage}, \text{carry-lr-wolf}, \text{carry-rl-wolf}, \text{cross-rl}, \text{cross-lr}, \text{eat-cabbage}, \text{eat-goat} \}\) is a set of actions.
  - (For instance:
    - \text{carry-lr-goat} means: the ferryman carries the goat from the left to the right river bank
    - \text{carry-rl-goat} means: the ferryman carries the goat from the right to the left river bank
    - \text{cross-rl} (resp. \text{cross-lr}) means: the ferryman crosses the river from right to left (left to right) without carrying anything.
    - \text{eat-cabbage} means: the goat eats the cabbage
    - \text{eat-goat} means: the wolf eats the goat.)

Assume that \(\text{Var} = \{ \text{goat}, \text{cabbage}, \text{wolf} \}\) and the corresponding domains are \(\{l, r\}\).

Let \(\text{Eval}(\text{Var}) = \{ \beta \mid \beta : \text{Var} \rightarrow \{l, r\} \}\).

- (Intuitively, \(\beta(x) = l\) means that \(x\) is on the left side of the river, and \(\beta(x) = r\) means that \(x\) is on the right side of the river.)

Assume that \(\text{Cond}(\text{Var}) = \{ \text{goat} \approx l, \text{goat} \approx r, \text{cabbage} \approx l, \text{cabbage} \approx r, \text{wolf} \approx l, \text{wolf} \approx r \}\)

and that the initial condition is

\[
g_0 := (\text{goat} \approx l) \land (\text{cabbage} \approx l) \land (\text{wolf} \approx l)
\]
(1) Define a suitable effect function $\text{Effect} : \text{Act} \times \text{Eval}(\text{Var}) \rightarrow \text{Eval}(\text{Var})$.

(It is not necessary to exhaustively present the definition of this function, you can present some examples and explain how it is defined in general)

(2) Define a suitable transition relation $\rightarrow \subseteq \text{Loc} \times (\text{Cond}(\text{Var}) \times \text{Act}) \times \text{Loc}$ such that there is no $\phi \in \text{Cond}(\text{Var}), \alpha \in \text{Act}, l \in \text{Loc}$ such that $(\text{conflict}, \phi, \alpha, l) \in \rightarrow$.

(It is not necessary to exhaustively present the definition of the transition relation $\rightarrow$; you can explain how it is defined in general and give some examples)

(3) Describe the transition system $TS(\text{PG}) = (S, \text{Act}, \rightarrow, I, \text{AP}, L)$ of the program graph $(\text{Loc}, \text{Act}, \text{Effect}, \rightarrow, \text{Loc}_0, g_0)$ constructed before.

(It is not necessary to exhaustively present the definition of the transition relation $\rightarrow$ or the labeling function; you can explain how they are defined in general and give some examples)

(4) Describe:

- $\text{Post}(\langle \text{left}, \beta \rangle, \text{carry-lr-goat})$, where $\beta(\text{goat}) = l, \beta(\text{cabbage}) = \beta(\text{wolf}) = r$.
- $\text{Post}(\langle \text{left}, \beta \rangle, \text{carry-rl-goat})$, where $\beta(\text{goat}) = l, \beta(\text{cabbage}) = \beta(\text{wolf}) = r$.
- $\text{Post}(\langle \text{left}, \beta \rangle)$, where $\beta(\text{goat}) = l, \beta(\text{cabbage}) = \beta(\text{wolf}) = r$.
- $\text{Post}(\langle \text{right}, \beta \rangle)$, where $\beta(\text{goat}) = \beta(\text{cabbage}) = l, \beta(\text{wolf}) = r$.
- $\text{Post}(\{\langle \text{right}, \beta \rangle, \langle \text{right}, \beta' \rangle\})$, where $\beta(\text{goat}) = \beta(\text{cabbage}) = l, \beta(\text{wolf}) = r$ and $\beta'(\text{goat}) = \beta(\text{wolf}) = l, \beta(\text{cabbage}) = r$
- $\text{Pre}(\langle \text{conflict}, \beta \rangle)$, where $\beta(\text{goat}) = \beta(\text{cabbage}) = l, \beta(\text{wolf}) = r$.
- $\text{Pre}(\langle \text{conflict}, \beta \rangle)$, where $\beta(\text{goat}) = \beta(\text{wolf}) = \beta(\text{cabbage}) = r$.

(5) Is the transition system you constructed action-deterministic? Is it $\text{AP}$-deterministic?

(7) Are there terminal states in the system?

(8) Is the state $\langle \text{right}, \beta \rangle$ with $\beta(\text{goat}) = \beta(\text{cabbage}) = \beta(\text{wolf}) = r$ reachable?

Please submit your solution until Tuesday, December 7, 2021 at 16:00. Please do not forget to write your name on your solution.

Submission possibilities:

- In directory Homework-06 in OLAT (preferred);
- By e-mail to sofronie@uni-koblenz.de with the keyword “Homework FSV” in the subject.