SAFE CORRIDORS FOR 
BACKWARD MOTION CONTROL OF TRUCK AND TRAILER

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ABSTRACT

A human being is able to drive backwards with truck and trailer. However, for a learner driver this constitutes one of the essential difficulties to cope with. In general a lot of practice combined with several rules are necessary to control the motion of the vehicle.

Backward motion of truck and trailer is a challenge in transport automation. Here two strategies are applied for solving this problem. One strategy, which can be called anthropomorphic, is to translate the driving rules for a human being into the code of the control algorithm. And the other one is to deal with this problem in an analytical manner. The first strategy is applied for controlling the vehicle when following its trajectory and the second strategy is applied for the derivation of safe corridors which are never left by the vehicle in motion.

Keywords: Transport automation, motion control, safe corridors

1. INTRODUCTION

The control theory distinguishes between different classes of vehicles with respect to their kinematics. The easiest class of vehicles is called holonomic and allows any change of direction. In contrast nonholonomic vehicles have a limited freedom of motion. This is already given for a single truck, also called standard 1-trailer [4]. The freedom of motion for standard n-trailers which are composed of a truck and n − 1 one-axle trailers, are described by chain-form differential equations and are investigated with regard to their controllability (see e.g. [3], [7],[8]). However, their approach is based on the geometric condition that the coupling device between truck and trailer is exactly above the backward axis of the truck. For standard transportation vehicles, as well for truck and trailer as for semi-trailers, there is a distance between these two positions. This is expressed with the notation of a general n-trailer [1] leading to a control problem which is even more challenging.

A possible context of application may be found in logistics centers. In these centers there already exists a high degree of automation with respect to disposition and dispatching of goods inside the building. However, outside in the yard truck and trailer are still driven manually to and from charging and discharging ramps.

The automation of the non-public traffic of a logistics center relies on motion control for standard transportation vehicles. In spite of a deeply investigated theoretic background there are only a few practical results for maneuvering realistic trucks and trailers available at the time (see [5], [9], and [2]). No steps so far have been done towards the integration of those maneuvering capabilities into the automation of transportation processes carried out by standard vehicles. This paper does a decisive step towards this direction by the development of safe corridors based on elementary cover segments.

The approach presented in this article is part of the project EZauto1 which tries to derive analytical and practical results in the context of autonomous driving, as cited above. A characteristic observation is the existence of a non-negligible difference between the perceived positions of the vehicle and the desired ones. Additionally there is a latency between the perception input and the control output. One of these outputs should be an emergency halt which should take effect before any unacceptable situation is reached. In general there exists the necessity for a predefined anticipation of all possible movements of the vehicle.

From the application oriented point of view, e.g. the autonomous maneuvering of trucks and trailers for charging and discharging purposes on the yard of some logistics center, a dispatch instruction initiates an automatic transportation operation which in more detail consists of:

1: the computation of a trajectory between different positions of the vehicle augmented by estimating the surrounding space for the execution of the maneuver with the constraints not to leave the yard and not to collide with any obstacle,

2: the execution of the control algorithm for the vehicle to follow the trajectory inside some predefined corridor,

3: the observation of the vehicles’ movements with the objective not to leave the estimated surrounding space, eventually executing an emergency halt.

Figure 1: The model-truck in operation

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1EZ is the acronym for the German word Echtzeit (real-time) and auto is an abbreviation which equally stands for autonomous and automotive.
Based on this rule experiments show that the ramp position can be reached with a precision of about ±10mm in x, y-direction and ±1° in the orientation of the trailer (see [11]).

Hence, the missing computations to achieve an application oriented solution are in (1.b) and (1.c). With respect to (1.c) the necessary test can be done efficiently by standard software if the geometry of the cover segments and the environment is based on elementary forms. So, it is to (1.b) to define those forms which will be elaborated in three subsequent steps:

1.a: computation of a feasible trajectory
1.b: computation of the cover segments for the trajectory
1.c: testing for conflict of the cover segments with the environment

The structure of trajectories for trucks with one-axle trailers has been discussed extensively (see [11] and [10]). Not considering limits for the yard or the existence of obstacles there is also a lot of experience with the control of model-truck at a scale of 1 : 16. The experimental system (see figure 1) operates with a control algorithm which adopts an anthropomorphic driving rule saying that

- the truck should follow the trailer and
- the trailer follows an imaginary circle.

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1.b.1: extreme points: certain positions of the truck and the trailer determine the necessary space of the vehicle pursuing the trajectory
1.b.2: covering segments: the test for conflict (1.c) demands for simple covers, therefore two elementary geometric forms are introduced covering the extreme points of truck and trailer
1.b.3: safe cover segments: due to technical and structural deficiencies a real vehicle deviates from the prescribed trajectory, therefore the covering segments are enlarged to a size guaranteeing that a vehicle never leaves this predefined area

This paper starts with the model of a vehicle and the basic trajectories which can be pursued by a truck with a one-axle trailer (section 2). In the sequel the concept of safe motion planning based on extreme curves (section 3), cover segments (section 4), and safe cover segments (section 5) is developed. The paper closes with the assessment of this approach, based on anthropomorphic and analytical strategies, in the scope of logistics centers (section 6).

2. TRAJECTORIES FOR TRUCKS WITH ONE-AXLE TRAILERS

There is a high degree of freedom for driving truck and trailer from a certain starting position to a final position. Ingenious ideas to reduce this degree of freedom also reduce complexity. So, the central idea is to maneuver truck and trailer along a trajectory based on polygons from a starting position to a final position.

More accurately the task is to apply a control algorithm is to make the entire vehicle follow a straight line and then turn to a subsequent line. Turning the vehicle means in detail to start a certain maneuver on a line with direction \( \phi_1 \) and to terminate this maneuver when the subsequent line with direction \( \phi_2 \) is reached (see figure 7).

Due to the nonholonomic properties of a truck with a one-axle trailer this can be done in a sequence of three well defined maneuver phases (for the notations of lengths and angles see figure 2 depicting the so called bike-model which reduces an axis to one imaginary wheel in its middle). The following explains these phases in detail for \( \phi_1 < \phi_2 \):

1-2 : The truck drives backwards on a left circle with the steering wheels at the constant angle \( \alpha = \alpha_{\text{max}} \). This phase of the maneuver starts with \( \gamma = 0 \) and lasts until \( \gamma = \gamma_{\text{circ}} \) which allows to proceed on a circle with a given radius \( r_{zk}(\alpha_{\text{circ}}) \) as described in formulas (1) and (2). Meanwhile the trailer turns right and changes its direction by an angle \( \Delta\phi_{1-2} \).

2-3 : Instantaneously the steering wheels are turned to the angle \( \alpha = -\alpha_{\text{circ}} \) for driving right on a sector of a circle with radius \( r_{zk}(-\alpha_{\text{circ}}) \). This phase whichs lasts until the trailer has changed its direction by the angle \( \Delta\phi_{2-3} \) all parts of the vehicle move on one circle with the same center.

3-4 : The angle \( \Delta\phi_{3-4} \) is gained by driving backwards in the last phase with the steering wheels at an angle \( \alpha = -\alpha_{\text{max}} \). This phase starts with \( \gamma = \gamma_{\text{circ}} \) and ends when the angle \( \gamma = 0 \) is reached, that is to say when the trailer is straight behind the truck again.

Two different types of curves are used for the maneuver. In phase 2-3 all points of the truck or trailer move on circles with the same center. In this case the angle \( \gamma \) between truck and trailer is a function of the steering angle \( \alpha \):

\[
\gamma(\alpha) = 180^\circ - \arccos\left(\frac{l_{zk} - l_{za}}{r_{zk}(\alpha)}\right) - \arccos\left(\frac{\eta_{aa}}{r_{zk}(\alpha)}\right)
\]

(1)

More difficult are the curves for maneuver phase 1-2 and phase 3-4 and the extraction of the relevant sections. The derivation of these curves is based on a modification of the classical tractrix problem. The point of traction for the trailer is the coupling device \( zk \) which moves on a circle during phase 1-2 and 3-4. This movement is proportional to angle \( u \) (see figure 2). As these phases are used to manipulate the angle between truck and trailer it is necessary to derive the function \( \gamma(u, \alpha) \) (see [9]):

\[
\gamma(u, \alpha) = \pi/2 - 2 \arctan\left(\frac{r_{za}(\alpha)}{r_{zk} - l_{aa} - \arctan\left(\frac{r_{za}(\alpha)}{r_{zk} - l_{aa}}\right)}\right) - \eta(\alpha)
\]

(2)

The movement of the trailer represented by its axis \( aa \) can easily be computed from \( \gamma(u, \alpha) \). This approach based on a special tractrix yields a set of closed formulas – in contrast to other approaches – for standard vehicles with one-axle trailer, including semi-trailers.
3. Extreme Points of the Vehicle in Motion

The bike model, as introduced in section 2, is no longer sufficient for the derivation of the surrounding space for a moving vehicle. In general the shape of a truck or a trailer is given by a rectangle. Even if there exist several axis with fixed wheels only one - possibly imaginary - axis with fixed wheels is assumed. For the truck there exists one axis with steering wheels at some angle \( \alpha \). For the trailer the front end of the shaft has the same function, given by angle \( \gamma \).

In the given context the cover is defined by a mapping of any point of a given shape of the truck or the trailer to the surface. Let \( \text{shape}(t) \) be the shadow of some vehicle at time \( t \), then the cover for some time interval \( [t_1, t_2] \) is:

\[
(x, y) \in \text{cover}(t_1, t_2) \Leftrightarrow \exists l, t_1 \leq t \leq t_2, (x, y) \in \text{shape}(t)
\]

The shape of the truck and the trailer depends on several groups of parameters:

- the configuration at some time \( t \) is the cartesian product \( C(t) = \bigtimes_k C_k(t) \) of position \( (x, y) \), the direction \( \beta \) of the truck and the angle \( \gamma \) between truck and trailer. By formula (2) these parameters depend on angle \( \alpha \) whereas for constant velocity of the truck \( \alpha \) has a linear dependency to \( t \). Thereby the configuration is defined to be a function of time.
- the kinematic parameters \( K \subseteq \mathbb{R}^3 \) describe the kinematic properties of a truck with a one-axle trailer given by \( lza, lzk, \) and \( laa \) (here \( lza = 600 \text{mm}, lzk = 600 \text{mm} \) and \( laa = 500 \text{mm} \)).
- the shape parameters \( S = \bigtimes_k S_k \) for truck and trailer, e.g. the breadth of the trailer.

For \( \alpha(t) = 0, t \in [t_1, t_2] \) the cover of the truck constitutes a rectangle. For other angles \( 0 < |\alpha(t)| \leq \alpha_{max} < 90^\circ \) the cover is rather irregular. At some time \( t \) the curvature at position \( za \), given by

\[
\pm \frac{1}{rz\alpha(t)} = \frac{lza}{\tan \alpha(t)}
\]  

determines \( \text{shape}(t) \). Positive curvature values indicate left curves and negative right curves.

Referring to the maneuver of the truck the curvature is constant in any phase. For the trailer it is only constant in phase 2-3 and steadily changing during phase 1-2 and phase 3-4. Furthermore the curvature jumps from phase to phase for truck and trailer (see figure 3).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{curvature.png}
\caption{Curvatures for the vehicle in positions \( zl \) and \( aa \) depending on the distance \( d \) done by the coupling device and henceforth proportional to time \( t \) in the case of constant velocity.}
\end{figure}

4. Cover Segments

For the vehicle to turn counterclockwise for some angle \( \Delta \phi = \phi_4 - \phi_1 \) the different phases of the necessary maneuver contribute different angles:

\[
\Delta \phi = \Delta \phi_{1-2} + \Delta \phi_{2-3} + \Delta \phi_{3-4}
\]  

The angles for our model-truck are: \( \Delta \phi_{1-2} = 3.84^\circ \) and \( \Delta \phi_{3-4} = 2.93^\circ \). The angle \( \Delta \phi_{2-3} \) is scalable to gain the desired increase of angle for the vehicle. During phase 2-3 the whole shape moves on a circle with a unique center \( c_{2-3} \). This leads to the idea to cover all extreme points

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{cover_segments.png}
\caption{The extreme points of truck and trailer}
\end{figure}
Figure 5: As a lower bound for the distance of $aar(u)$ to $c_{2-3}$ during phase 1-2 the value $(*)$ is set to the height of the triangle given by the points $aar(u1)$, $aar(u2)$ and $c_{2-3}$.

Figure 6: The positions $zk$ and $aa$ of the model-truck following a precomputed trajectory.

### 5. Safe Cover Segments

Up to this point the discussion has been purely analytical and describes the ideal behavior of a virtual vehicle. The real model-truck represents a considerable value which has to be protected against damage caused by itself. Furthermore there is a real environment with real I/O devices and a control algorithm which has to cope with inaccuracies due to different reasons.

First of all there exists a control algorithm based on anthropomorphic rules for adjusting the whole vehicle to the precomputed trajectory. The results of this algorithm are satisfactory in that the deviations at end points of trajectories are rather low (see figure 6). However, due to the combination of heuristics it is impossible to give an adequate estimation of the maximal deviation from the trajectory in between (see also [6]).

### Table: Cover Segments

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Phase</th>
<th>$rc_{\text{min}}$</th>
<th>$rc_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truck</td>
<td>1-2</td>
<td>final points</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2-3</td>
<td>$zar$ $zvl$ or $zbl$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3-4</td>
<td>final points</td>
<td></td>
</tr>
<tr>
<td>Trailer</td>
<td>1-2</td>
<td>$(*)_1$</td>
<td>$(*)_2$</td>
</tr>
<tr>
<td></td>
<td>2-3</td>
<td>$aar$ $avl$ or $abl$</td>
<td>$(*)_3$</td>
</tr>
<tr>
<td></td>
<td>3-4</td>
<td>$aar$</td>
<td>$(*)_4$</td>
</tr>
</tbody>
</table>

Sometimes the extreme values of extreme points are constant during a phase, sometimes the minimal or maximal values are reached at the beginning or the end of a phase (final points in the table) and sometimes values are dominated (minimized or maximized) by values of some other column. However, there are also some sophisticated computations to determine or to bound $rc_{\text{max}}$ and $rc_{\text{min}}$. Exemplary the bounding of $(*)_2$ is described here. It reflects the movement of extreme point $aar$ of the trailer during phase 1-2. As the curvature at the end of phase 1-2 is higher than that of the subsequent circle (see circle 1 in figure 3) it follows that these points are nearer to the center $c_{2-3}$ than those of phase 2-3. To avoid the numerical computation of a minimal distance, the bounding value due to the construction in figure 5 is used instead as the minimal value $(*)_2$.

To sum up for the model-truck the values for the cover segment are the following: $\zeta_1 = 148.8^\circ$, $\zeta_4 = 212.8^\circ$, $rc_{\text{max}} = 2613\text{mm}$, and $rc_{\text{min}} = 2075\text{mm}$ (see also the cover segment in figure 9).
The control algorithm's principle concern in this section is to derive the minimal applications and may be found by a series of tests. The whole control system is time triggered and the control algorithm executes in a major loop. For loop i the sensor input $SI_i$ and the reference input $RI_i$ are the basis to compute the actuator output $AO_i$:

$$AO_i = ca(SI_i, RI_i)$$

The sensor data $SI_i$ constitute an internal image of the technical system in progress. When $SI_i$ is used by algorithm $ca$ at time $t_{ca}$ it already has an age of at least $\Delta t_p$, since being sampled by some input device. Additionally, the values $SI_i$ are only some approximation of the real configuration $C(t_{ca} - \Delta t_p)$ at that time. An interval $[c_{j,i}, c_{j,i}]$ is associated with any configuration parameter $C_j(t_{ca} - \Delta t_p), j \in \{1, ..., k\}$ indicating minimal and maximal configuration parameters which are capable at that time to produce the input sample $SI_i$.

During the same control loop $i$ it has to be decided whether to execute an emergency halt or not. Let $\Delta t_h$ be the necessary duration for stopping the vehicle. At time $t_{ca}$ the decision for an emergency halt has to be made when subsets of $shape(t)$ leave the corridor of safe cover segments in the time interval $[t_{ca} - \Delta t_p, t_{ca} + \Delta t_h]$. As the computation of $shape(t)$ depends on the configuration at time $t_{ca} - \Delta t_p$ all possible configurations should be considered. In a straightforward manner the algorithm would read as follows:

1. take a relevant configuration $c \in \times_k C_k(t_{ca} - \Delta t_p)$, under certain monotonic conditions it may be sufficient to select $c \in \times_k c_{j,i} \times c_{j,i}$
2. compute $c(t)$ based on the formula 2 for $t \in [t_{ca} - \Delta t_p, t_{ca} + \Delta t_h]$
3. from $c(t)$ construct $shape(t)$
4. based on $shape(t)$ the cover $(t_{ca} - \Delta t_p, t_{ca} + \Delta t_h)$ has to be constructed
5. trigger an emergency halt if cover $(t_{ca} - \Delta t_p, t_{ca} + \Delta t_h)$ exists outside the corridor of safe cover segments

There are various heuristics (e.g. the construction of the cover) missing for an efficient realization in this straightforward manner. Furthermore, the decision has to be taken under hard real time conditions in a few milliseconds. Therefore this algorithm has to be replaced by far simpler assumptions about the real behavior of the vehicle in the interval $[t_{ca} - \Delta t_p, t_{ca} + \Delta t_h]$. This is done in a rather pragmatic fashion by considering the particular properties of the given model-truck. Equally it is not omitted to have a look at real trucks and trailers.

- When driving backwards the velocity taken at the fixed axis $za$ of the truck is at about 60 - 70mm/s, never reaching $vza_{max} = 80mm/sec$. Compared to a real truck this corresponds to a velocity of about 1m/s which is rather fast when driving backwards.
- The steering angle $\alpha$ is limited to 30° by the mechanics of the model-truck.
- The angle $\gamma$ between truck and trailer is limited to 25° by the length of the shaft and the breadth of the vehicle. When reaching this angle an emergency halt is triggered also.
- The model-truck (like standard trucks) has standard dimensions, that is to say that $lza = 1.5$ is longer than the distance from the fixed axis of the truck to any point of the truck.

The model-trailer (like standard trailers) has standard dimensions, that is to say that $laa$ is longer than the distance from axis of the trailer to any other point of the trailer.

There is a standard relation between truck and trailer, that is to say that $lzk > laa$.

The essence of this coarse estimation is that no point $z$ of the truck ever moves faster than two times the maximal: $v_{sls} < 2vza_{max}$. With the time bound $\Delta t_p + \Delta t_h < 550ms$ the maximum distance for the perception of a critical situation and the subsequent emergency halt is less than 90mm. Putting it the other way any margin $> 90mm$ may be considered to find a good compromise between wasting maneuver space and boring zigzag movements of the vehicle.

With respect to a real truck the margin of 90mm corresponds to 1.44m surrounding the cover segment. The cover segment itself has a breadth of 8.608m and henceforth the safe cover segment a breadth of 11.488m. This seems to be rather much compared to the assumed breadth of 3.20m for equally truck and trailer. However, it has to be considered that the safe cover segment really captures all maneuvers and deviations at a speed of up to 1m/s.

For the construction of entire corridors a simple rule for the connection of subsequent safe cover segments has to be applied. The points of connection correspond to the points of starting and ending of the maneuver at certain distances from the intersecting lines of the polygon (see [10]) and their direction is either given by the polygon or an arc around the center of the circle used for phase 2-3 (see figure 10).

6. CONCLUSIONS

The model-truck constitutes a prototypical solution to the problem of transport automation logistics centers. Even if the scope of application is restricted to the non-public traffic on the yard of some logistics center various aspects of safety have to be considered with care.

One essential aspect of safety is to limit the space for the vehicle in motion. Guaranteeing that the vehicle never leaves a predefined corridor constitutes a valuable property for entire process of motion control and trajectory planning. Equally, it seems so be a valuable property that this corridor is composed of safe cover segments which are of elementary forms, particularly with respect to the idea to intersect these segments with other geometric forms which constitute the limits of the yard or represent some obstacle.

Furthermore the concept of safe cover segments may be used for the dynamic scheduling of several vehicles on the same yard. Based on the invariant property that at a time a vehicle occupies only one safe cover segment, the problem of collision avoidance can be reduced to a schedule of non-intersecting segments. This allows for a high density of traffic on the yard of a logistics center.

On one hand the approach presented here is based both on analytical derivations for closed formulas of trajectories between two points on the yard and the construction of safe cover segments for the distinct parts of the trajectory. On the other hand the strategy for controlling the motion of the vehicle is based on anthropomorphic driving rules. A disadvantage of the latter is that there are no analytical bounds for the degree of deviation for this control algorithm. Due to this lack the concept of safe cover segments entirely relies on emergency halts. Instead, in presence of narrow bounds for the control algorithm these bounds should be used for the construction of an other kind of safe cover segments which allow for maximal unidirectional movements without any annoying zigzag.
standard transportation vehicle has not been considered here. This is the truck with a two-axle trailer in the metric of holonomy also called \textit{general 3-trailer}. Practical results with this vehicle are at a very early state (see [1]). Preliminary experiments and deliberations in the project EZauto allow for the conjecture that a specific set of anthropomorphic driving rules may lead to an acceptable algorithm for controlling the motion of the vehicle following a pre-defined trajectory. However, by now no closed formulas for the decisive maneuver phases are available.

REFERENCES


7. APPENDIX

The following sequence of figures shows the decisive steps to develop safe corridors for a standard situation: gaining some angle (here 22.5°) for truck and trailer.

Figure 7: Reference trajectory for position $aa$ of the trailer to change from direction $\phi_1$ to $\phi_4$.

Figure 8: The states 1-4 of the maneuver with corresponding steering angles.

Figure 9: Three types of covers: the extreme points of the moving vehicle, the cover segment and, the save cover segment.

Figure 10: The construction of a corridor with safe cover segments.